PART 1

INTRODUCTION TO DERIVATIVES AND RISK MANAGEMENT

CHAPTER 1

An Overview of Derivative Contracts

A derivative is a financial contract whose value is "derived from," or depends on, the price of some underlying asset. Equivalently, the value of a derivative changes when there is a change in the price of an underlying related asset. This chapter provides a broad overview of the four classes of derivative contracts: forwards, futures, swaps, and options. However, because forwards, futures, and swaps are very similar types of contract, many believe that there are really only two types of derivative: options and forwards. While other derivative contracts exist, a careful analysis of their characteristics will reveal that they are merely variations of one (or more) of these major classes.

Mark Rubinstein (1976) was perhaps the earliest author to use "derivative" in a financial contract context; in this particular article, he used the term to refer to options. A computerized search found that it wasn't until 1981 that an article in the popular press *Business Week* (1981) used the term, with quotation marks around it. This article expressed the fear that trading in options and futures (derivatives) detrimentally affects the markets of the underlying assets. It also raised the issue of who—the Securities and Exchange Commission (SEC) or the Commodity Futures Trading Commission (CFTC)—should regulate trading in derivatives. Currently, "derivatives" is routinely used in business communication and all forms of media, and textbooks and college courses are devoted to the study of the subject.

Individuals of many types use derivatives. Speculators who think they know the future direction of prices use derivatives to try to profit from their beliefs. Arbitrageurs trade derivatives to take advantage of times during which prices are "out of sync": that is, one asset or derivative contract is mispriced relative to another. Hedgers face the risk that a change in a price will hurt their financial status; they use derivatives to protect, hedge, or insure themselves against such harmful movement in prices. Of particular interest is the current indispensability of derivatives for accomplishing many tasks necessary to the successful management of corporations, governments, and large pools of money in general: managing exposure to price risk, lowering interest expense, altering the structure of assets, liabilities, revenues, and costs, reducing taxes, circumventing unwieldy regulations that make transactions difficult, and arbitraging price differentials.

We can characterize derivatives by the structures of the markets in which they trade. Some derivatives trade on organized exchanges. In particular, there are options and futures exchanges in existence all over the world.[1] These exchanges allow virtually anyone who meets some set of financial criteria (e.g., the individual's net worth or income) to trade these contracts. Price and trade information are readily available, and at any point in time, the prices at which they can be bought do not appreciably differ from the prices at which they can be sold. Other derivative contracts actively trade in liquid and well-established over-the-counter (OTC) markets. These markets are open only to large, financially sound corporations, governments, and other institutions.

Large, well-capitalized financial institutions such as Morgan Stanley Dean Witter, Goldman Sachs, and Deutsche Bank quote prices for these contracts; the bid–ask quotes are characterized by having narrow spreads, and trading is often quite active. Finally, many derivatives are custom made by these financial institutions for a specific end user. Generally, the party desiring such unique derivative contracts enters into the agreement with the intent of keeping the position until it matures. In general, all derivatives that do not trade on an exchange are called OTC contracts.

For many assets, the size of the derivatives markets is many times larger than the size of the cash market. Indeed, 3 trillion futures and options contracts traded on organized exchanges in 2000, an increase of over 24% from 1999 (Burghardt, 2001, p. 34). The Bank for International Settlements (BIS) reported in its November 13, 2000, press release (www.bis.org/) that the total notional principal[2] of outstanding OTC derivatives as of June 30, 2000 was $94 *trillion*! The sizes of the individual markets will be noted in the following overviews of the four different types of derivative. More detailed analyses of the contracts are contained in subsequent chapters.

1.1 Forward Contracts and Futures Contracts

A forward contract gives the owner the right and obligation to buy a specified asset on a specified date at a specified price. The seller of the contract has the right and obligation to sell the asset on the date for that specified price. At delivery, ownership of the good is transferred and payment is made. In other words, whereas the forward contract is executed today, and the price is agreed upon today, the actual transaction in which the underlying asset is traded does not take place until a later date.

Typically, no money changes hands on the origination date of a forward contract. However, because forwards contain a promise to exchange the good for cash at a later date, each party wants some form of assurance that the counterparty will strictly observe the terms of the contract. Therefore, it is not unusual for the more creditworthy party to demand that the less creditworthy party put up some collateral, or take other actions to reduce the possibility of default risk.

You have probably engaged in many forward transactions in your life. For example, if you call your barber to set up an appointment for a haircut, you have transacted in a forward market. Many major purchases of items such as cars are forward transactions, and sometimes, the seller will ask for a down payment. Usually, in these informal forward contracts, a party will not take legal action if the other party defaults. Imagine how you would feel if you made a reservation at a restaurant (which is a forward contract to purchase a meal, and pay for it, at a later date), failed to appear on time (or at all), and the restaurant brought legal action against you. On the other hand, many hotels will charge you for a room if you fail to show up on time after "taking on a long position in a forward contract" that obligates you to buy the use of a room for the period specified.

Defaults on derivatives, including forwards, are taken very seriously in the financial markets. A party that loses because its counterparty defaulted will typically take legal action. Furthermore, the defaulting party's reputation will be tarnished, likely making it difficult or impossible for the defaulter to engage in future derivative transactions. It is interesting to note that only one party at any time will have the incentive to default: the party that is losing as of that date. It would most certainly *not* be in the best interest of the other party, which has unrealized profits on its positions, to walk away from its obligations, since by doing so it would be giving up its rights to a likely future cash inflow.

A **futures contract** is similar to a forward contract; the differences will be presented in Chapter 6. Until then, think of a futures contract as a standardized forward contract[3] that can be easily traded. Futures and forwards both obligate the buyer (the party with a long position in the contract) to take delivery of the underlying asset at a future date and to pay for it upon delivery; both contracts give the seller the obligation to deliver the good, asset, or service at a future date. Money typically changes hands on the delivery date.

Default risk is lower on futures than on forwards for several reasons: (a) the counterparty to all futures trades is actually the clearinghouse of the futures exchange, which guarantees that all payments will be made; (b) futures contracts are marked to market daily (daily resettlement), which means that any change in the value of the contract is realized as a profit or loss every day; and (c) initial margin, which serves as a performance bond, is required when trading futures. In contrast, because they are not marked to market, forward contracts can build up large unrealized profits for one party and equally large unrealized losses for the other party.

For example, suppose that today you sell a *forward* contract that commits you to sell one barrel of oil at $21/barrel. Then on each of the next five days, the forward price rises by $1/barrel per day. After five days, you have an unrealized loss of $5. Had you instead sold a *futures* contract, you would have been forced to pay $1 each day to the holder of the long position in this contract. In other words, the futures contract is settled up daily. Profits and losses are realized each day in the futures market, but they are realized only upon delivery in the forward market.

Because they are marked to market daily, a futures contract is like a portfolio (a time series) of forward contracts. Put another way, a futures contract with a delivery date one month (30 days) hence is equivalent to 30 forward contracts. The first forward contract originates today and is cash settled on its delivery date, which is tomorrow. The second contract originates tomorrow, and is cash settled the day after that. The last (the 30th) forward contract originates 29 days hence, and its delivery date is 30 days hence.

The profit diagrams for forwards and futures are identical. If the spot price of the underlying asset on the delivery date is above the forward price that was agreed upon when the contract was originated, the party who is long profits. All derivatives are zero-sum games. Whatever amount of money one party gains must equal the amount of money the other party loses. Alternatively, you can think in terms of changes in the forward price. Suppose that today is June 3, 2000, and today's forward price for buying gold on February 3, 2001, is $285/oz. Then, suppose on December 3, 2000, the forward price for buying gold on February 3, 2001 is $245/oz. The party that is short the forward contract has a profit of $40/oz. (this profit will still be unrealized as of December 3), and the party that is long the forward contract has a loss of $40/oz. These concepts are illustrated in Figure 1.1.

The concept of forward delivery, with contracts stating what is to be delivered for a fixed price at a specified place on a specified date, existed in ancient Greece and Rome.[4] Perhaps the first organized commodity exchange on which forward contracts traded was doing business in Japan in the early 1700s. The first formal commodities exchange in the United States for spot and forward contracting was formed in 1848: the Chicago Board of Trade (CBOT). Futures contracts began trading on the CBOT in the 1860s. The Chicago Mercantile Exchange (CME) was formed in 1919, though it essentially existed before that date under other names.

In an article published four years before he and two other Americans were awarded the 1990 Nobel Prize in economic science, Merton Miller wrote, "my nomination for the most significant financial innovation of the last twenty years is: financial futures—the futures exchange style trading of financial instruments" (Miller, 1986, p. 463).

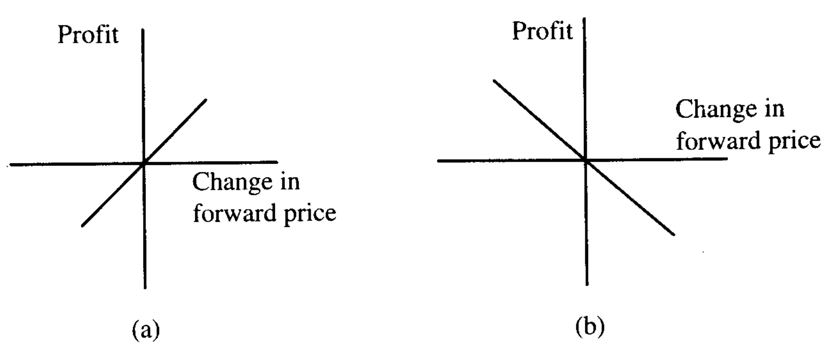

Figure 1.1 Profit diagrams for (*a*) a long forward position and (*b*) a short forward position.

The first financial futures contracts were the foreign currency futures contracts that began trading on May 16, 1972, on the International Monetary Market (IMM), a division of the CME[5]; 400 contracts were traded on the first day of operations. The first interest rate futures contracts that traded were the GNMA (Government National Mortgage Association) futures contracts,[6] on the CBOT, on October 20, 1975. The Eurodollar Time Deposit futures contract was the first "cash settled"[7] contract, and it began trading on the IMM on December 9, 1981. Another significant innovation occurred on February 24, 1982, when futures contracts on a stock index, the Value Line Index, began trading on the Kansas City Board of Trade; 1768 contracts traded on its opening day.

On September 7, 1984, the Singapore International Monetary Exchange (SIMEX) and the CME formed a trading link that permits investors to trade some contracts interchangeably on the two exchanges (this is called "mutual offset"). This was the first of several networks and systems that are leading to a globalized and increasingly automated world of 24-hour-a-day trading of securities.

Trading in energy futures commenced in the late 1970s and is now one of the most heavily traded sectors in the futures market. The New York Mercantile Exchange (NYMEX) introduced trading in No. 2 heating oil futures in November 1978. Gasoline trading began in 1981 and crude oil trading in 1983. Energy futures trade on many exchanges all over the world.

Figure 1.2 depicts how futures trading in the United States has exploded since the late 1960s. Much of the increase from 14.6 million contracts in 1971 to over 400 million contracts in 1994 and 1995 came from financial futures contracts, including futures on interest rate instruments, stock indexes, and foreign exchange. In 2000, 491.5 million futures contracts traded in the United States and another 952.6 million futures contracts traded outside the country (Burghardt, 2001). The notional principal of all the exchange traded derivatives (futures and options) traded in the world in 2000 was estimated to be $383 trillion (see the March 2001 BIS Quarterly Review, which can be viewed at www.bis.org/publ/); this was up 9.4% from the 1999 figure. Table 1.1 presents the actual number of futures contracts traded in the United States in several recent years. Table 1.2 breaks down volume in the recent past by the U.S. exchange on which the contracts traded.

It is interesting that futures trading, expressed in terms of number of contracts traded, peaked in 1998, followed by drops in 1999 and 2000. This occurred for two reasons. First, 1998 was a chaotic year in which Russia effectively defaulted on its debt and the currencies of many emerging market nations plummeted in value. These events led to considerable derivatives trading in general

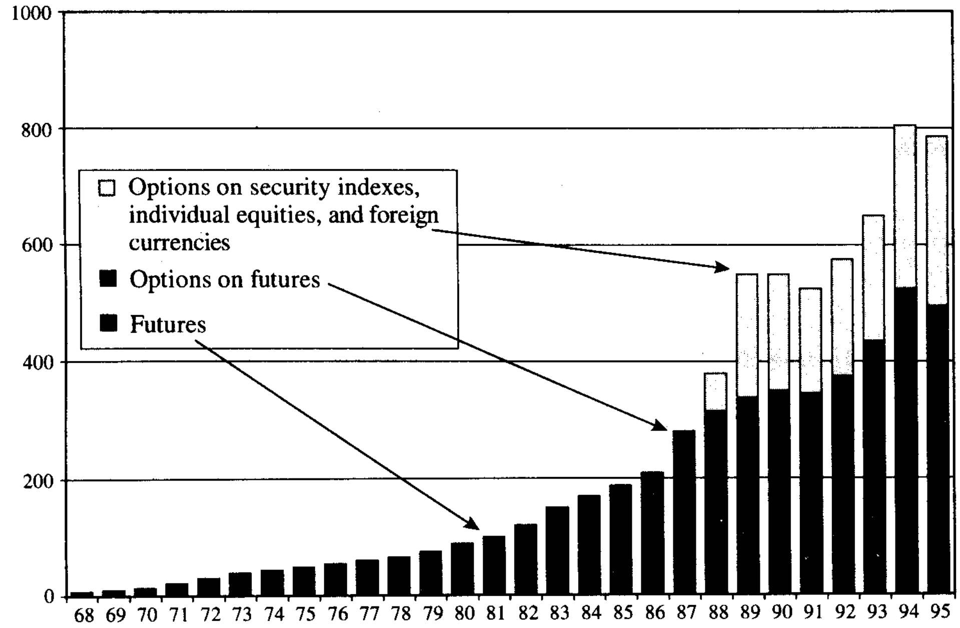

Figure 1.2 Volume of futures and options trading on U.S. futures and securities exchanges, 1968–1995.
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TABLE 1.1 Average Estimated Number of Futures Contracts Traded, All U.S. Markets Combined: FY 1990–FY 2000

Fiscal Year	Total Contracts Traded
1990	272,306,699
1991	261,422,699
1992	289,453,855
1993	325,515,261
1994	411,056,929
1995	409,420,426
1996	394,182,422
1997	417,341,601
1998	500,562,510
1999	491,137,790
2000	477,834,609

Source: CFTC website at www.cftc.gov/.

in 1998. Second, 1999 and 2000 were much calmer years. Also, the euro was not a traded currency in 1998, so in 1998 there was trading in French francs, German marks, Italian lira, and so on. All the trading in the different "Euroland" currencies consolidated into euro-trading in 1999.

Futures trading takes place on a huge scale internationally. Table 1.3 summarizes trading activity for the most active exchange-traded futures and options contracts traded in the world

TABLE 1.2 Estimated Number of Futures Contracts Traded in U.S. Markets for Fiscal Years Ending 9/30/99 and 9/30/00

Exchange	Volume of Trading (number of contracts)		Underlying Asset of Most Actively Traded Contracts
	1998–1999	**1999–2000**	**(1999–2000 volume)**
Chicago Board of Trade (CBT)	203,662,672	187,048,113	U.S. Treasury bonds 67,008,924 contracts U.S. 10-year Treasury notes 42,769,912 contracts
Kansas City Board of Trade (KCBT)	2,378,920	2,345,727	Wheat 2,324,744 contracts
Minneapolis Grain Exchange (MGE)	1,180,573	967,504	Wheat 963,054 contracts
MidAmerica Commodity Exchange (MCE)	2,586,219	1,913,154	U.S. Treasury bonds 763,334 contracts
Chicago Mercantile Exchange (CME) and International Monetary Market (IMM)	175,488,669	181,852,421	Three-month Eurodollars 100,452,601 contracts
New York Mercantile Exchange (NYMEX) and Commodity Exchange Inc. (COMEX)	89,869,532	87,753,321	Crude oil, light "sweet" 37,526,345 contracts
New York Board of Trade (NYBT), New York Cotton Exchange (NYCE), New York Futures Exchange (NYFE), Coffee, Sugar & Cocoa Exchange (CS&CE), and Cantor Exchange (CFFE)	16,070,698	15,954,369	Sugar no. 11 5,819,141 contracts
Total, all markets	491,137,790	477,834,609	

Source: CFTC website at www.cftc.gov/.

during the first eight months of 2000. Table 1.4, which lists volume figures for the leading international futures exchanges during 2000, illustrates the international importance of futures trading.

1.2 Swaps

In a swap contract, two parties agree to exchange cash flows at future dates according to a prearranged formula. Like a futures contract, a swap is also equivalent to a portfolio of forward contracts. However, whereas a futures contract is akin to a time series of one-day forwards, a swap is like a portfolio of forwards, all of which originate today and all of which have different delivery dates. Figure 1.3a illustrates the difference, using a futures contract with a delivery day (at time 4) four days after origination (the origination day is on day 0), and a four-period swap.

TABLE 1.3 Most Actively Traded Futures and Options Contracts for the Eight-Month Period Ending August 30, 2000

Contract	Exchange	Volume (number of contracts) January 1, 2000–August 31, 2000
Euro-BUND	EUREX-Frankfurt	105,736,279
KOSPI 200 Options	KSE, Korea	96,153,236
Eurodollars	CME, U.S.	72,920,253
CAC 40 Index Options	MONEP, France	54,795,638
U.S. Treasury Bonds	CBOT, U.S.	45,006,113
Euro-BOBL	EUREX-Frankfort	42,061,463
Three-month Euribor	LIFFE, UK	38,689,860
Euro-notional bond	Euronext Paris, France	33,049,313
U.S. T-bond options	CBOT, U.S.	31,270,050
Euro-SCHATZ	EUREX-Frankfurt	26,927,236

The exchanges are as follows: CBOT, Chicago Board of Trade (www.cbot.com); CME, Chicago Mercantile Exchange (www.cme.com); EUREX, Eurex (www.eurexchange.com/); KSE, Korea Stock Exchange (www.asiadragons.com/korea/finance/stock_market/); MONEP, Marché des Options Negociables de Paris (www.monep.fr/english/defaultuk.html); Euronext Paris (www.euronext.com); LIFFE, London International Financial Futures and Options Exchange (www.liffe.com).

Source: © October/November 2000, *Futures Industry*. Adapted with permission.

TABLE 1.4 Top Ten International Futures Exchanges in Terms of Number of Contracts Traded in 2000

Exchange	1999	2000
EUREX (Germany and Switzerland)	244,686,104	289,952,183
Chicago Mercantile Exchange (CME, U.S.)	168,013,354	195,106,383
Chicago Board of Trade (CBOT, U.S.)	195,147,279	189,662,407
London International Financial Futures and Options Exchange (LIFFE, U.K.)	97,689,714	105,712,717
New York Mercantile Exchange (NYMEX, U.S.)	92,415,006	86,087,640
Bolsa de Mercadorias & Futuros (BM&F, Brazil)	52,797,466	80,073,865
Paris Bourse SA (France)	35,129,074	62,968,653
London Metal Exchange (LME, U.K.)	57,563,009	61,413,076
Tokyo Commodity Exchange (Japan)	48,442,161	50,851,882
Euronext Brussels Derivatives Market (Belgium)	5,711,482	30,299,351

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A simple, plain vanilla, interest rate swap illustrates how a formula is applied to the notional principal[8] amount to determine the cash flows that are paid by one party to the other. In this swap, I agree to pay you 8% of $40 million each year for the next five years. You agree to pay me whatever one-year LIBOR is (times $40 million) for each of the next five years.[9] The net payments are therefore:

$$\text{if LIBOR} > 8\%, \text{ you pay me (LIBOR} - 8\%) \times \$40 \text{ million}$$
$$\text{if LIBOR} < 8\%, \text{ I pay you } (8\% - \text{LIBOR}) \times \$40 \text{ million}$$

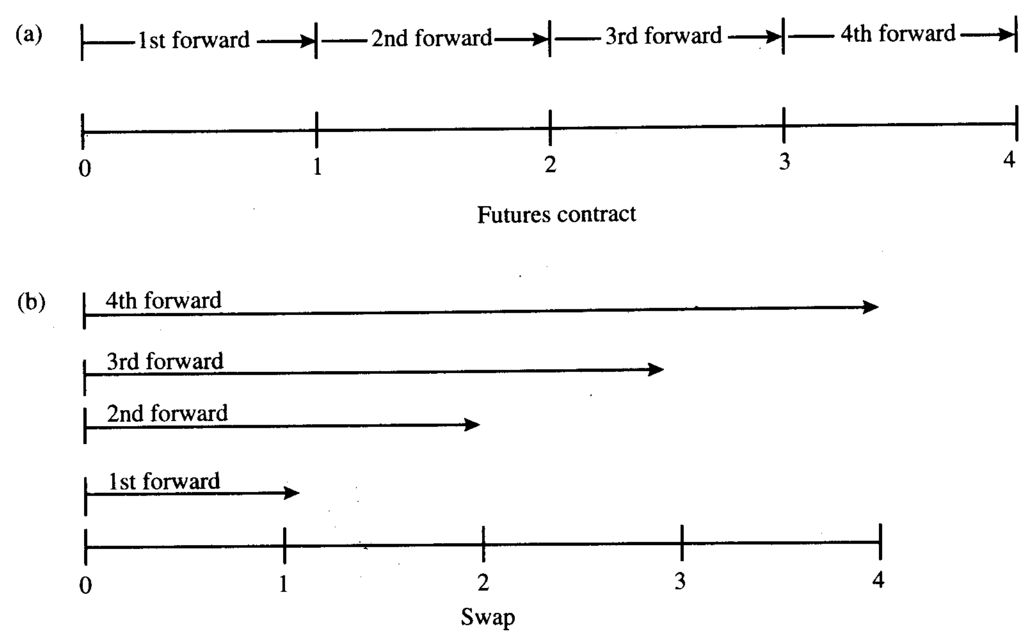

Figure 1.3 (*a*) A futures contract is like a time series of one-day forward contracts. (*b*) A swap is like a portfolio of forward contracts, all of which originate on the swap origination day (day 0), and all of which have different delivery dates.

Equivalently, I am long forward rate agreements, with delivery dates at the end of each of the next five years. As long as LIBOR is above 8%, I will profit.

The size of the swaps market has grown enormously since the first swap was transacted 1976. Continental Illinois, Ltd. and Goldman, Sachs arranged that first swap, which was a currency swap, between a Dutch company (Bos Kalis Westminster) and a British firm (ICI Finance). The currencies swapped were Dutch florins and British pounds. The first interest rate swaps took place in 1981.[10]

The growth in the swaps market is illustrated in Table 1.5, which shows that in terms of notional principal, interest rate swap trading activity expanded from $387.8 billion in 1987 to $17.1 trillion in 1997. The notional principal of interest rate swaps outstanding has grown from $682.8 billion at the end of 1987 to $48 trillion as of June 30, 2000. Trading activity of currency swaps grew from $85.8 billion in 1987 to $1.1 trillion in 1997, while the notional principal outstanding of currency swaps grew from $182.8 billion at the end of 1987 to $2.605 trillion in June 2000. The Bank for International Settlements (BIS) reported that outstanding OTC derivatives, including swaps, forwards, and OTC options, had grown to $94 trillion in notional principal on June 30, 2000 (see the press release at www.bis.org/publ/).

Table 1.5 also illustrates the expansion in the OTC interest rate options market. The amount outstanding increased from $327.3 billion to almost $8 trillion between the end of 1988 and the end of 1998. The BIS reports the size of the entire (foreign exchange, interest rates, equity-linked, and commodity) OTC options market to be $13.2 trillion as of June 30, 2000 (www.bis.org).

TABLE 1.5 Swaps: Annual Activity and Outstanding (U.S.$ billions of notional principal)

	Interest Rate Swaps		Currency Swaps		Interest Rate Options		Totals	
Year Ending	**Activity**	**Outstandings**	**Activity**	**Outstandings**	**Activity**	**Outstandings**	**Activity**	**Outstandings**
1987	$387.8	$682.80	$85.8	$182.80			$473.6	$865.6
1988	$568.1	$1,010.20	$122.6	$316.80		$327.30	$690.7	$1,654.3
1989	$833.6	$1,502.60	$169.6	$434.80	$335.5	$537.30	$1,338.7	$2,474.7
1990	$1,264.3	$2,311.50	$212.7	$577.50	$292.3	$561.30	$1,769.3	$3,450.3
1991	$1,621.8	$3,065.10	$328.4	$807.20	$382.7	$577.20	$2,332.9	$4,449.5
1992	$2,822.6	$3,850.80	$301.9	$860.40	$592.4	$634.50	$3,716.9	$5,345.7
1993	$4,104.6	$6,177.30	$295.2	$899.60	$1,117.0	$1,397.60	$5,516.8	$8,474.5
1994	$6,240.9	$8,815.60	$379.3	$914.80	$1,513.2	$1,572.80	$8,133.4	$11,303.2
1995	$8,698.8	$12,810.70	$454.1	$1,197.40	$2,015.4	$3,704.50	$11,169.3	$17,712.6
1996	$13,678.2	$19,170.90	$759.1	$1,559.60	$3,337.2	$4,722.60	$17,774.5	$25,453.1
1997	$17,067.1	$22,291.30	$1,135.4	$1,823.60	$3,978.4	$4,920.10	$22,180.9	$29,035.0
1998		$36,262.00		$2,253.00		$7,997.00		$46,512.0
1999		$43,936.00		$2,444.00		$9,380.00		$55,760.0

Sources: Reprinted with permission of the International Swaps and Derivatives Association, Inc. (ISDA) (www.isda.org/d6.html) from sources including the Bank for International Settlements (BIS) (www.bis.org/publ/).

TABLE 1.6 The Global OTC Interest Rate Derivatives Market Amounts Outstanding (billions of U.S. dollars)[1]

	By Currency		
	December 31, 1998	**December 31, 1999**	**June 30, 2000**
U.S. dollar	$13,763	$16,510	$17,606
Euro	$16,461	$20,692	$22,948
Japanese yen	$9,763	$12,391	$12,391
Pound sterling	$3,911	$4,588	$4,741
Swiss franc	$1,320	$1,414	$1,409
Canadian dollar	$747	$825	$846
Swedish krona	$939	$1,373	$1,255
Other	$3,113	$2,298	$2,558
	By Maturity[2]		
Up to one year	$18,185	$24,874	$25,809
Between one and five years	$21,405	$23,179	$24,406
Over five years	$10,420	$12,038	$13,910
Total contracts	$50,015	$60,091	$64,125

[1] All figures are adjusted for double-counting. Notional amounts outstanding have been adjusted by halving positions vis-à-vis other reporting dealers.
[2] Residual maturity.
Source: Press release dated November 13, 2000 (www.bis.org). Reprinted with permission of the Bank for International Settlements.

A more detailed breakdown, by currency and maturity, of recent OTC derivatives activity is provided in Tables 1.6 and 1.7. Table 1.6 presents the notional amount outstanding of interest rate derivatives, most of which are interest rate swaps. Table 1.7 shows the same information for foreign exchange derivatives, most of which are currency swaps. The tables show that interest rate derivatives involving the euro (i.e., the formulas were expressed in terms of euro-denominated interest rates, and all cash flows were in euros) is the largest market, with U.S.$22.9 trillion amount outstanding as of June 30, 2000. Another $17.6 trillion of OTC interest rate derivatives involved the U.S. dollar. The third most popular currency for interest rate derivatives is the Japanese yen. Foreign exchange derivatives involving the U.S. dollar (and some other currency) comprise the largest market in terms of currencies. The size of the U.S. dollar foreign exchange derivatives market, in terms of notional principal amount outstanding, was $14 trillion, which was 45.1% of the total market.

Other tables in the BIS report show that 40.2% of all interest rate derivatives had a maturity of less than one year, and another 38.1% matured in one to five years. Eighty-five percent of all outstanding currency swaps had a maturity of less than one year.

The BIS data indicated that the total notional principal of outstanding privately negotiated (OTC) interest rate swaps and currency swaps as of June 30, 2000 stood at $50.6 trillion.

Swap dealers have regretted that they express the sizes of their markets in terms of notional principal. The regret arises when government legislators and others voice alarm at the size of this unregulated (by the U.S. government) market. Upon hearing the size of this market: $94 trillion (in notional principal), a naïve person would fret over what would happen to the world's financial system if trading abuses and/or massive defaults occurred. One party might not abide by the terms

TABLE 1.7 The Global OTC Foreign Exchange Derivatives Market Amounts Outstanding in Billions of U.S. dollars[1,2]

	By Currency		
	December 31, 1998	**December 31, 1999**	**June 30, 2000**
U.S. dollar	$15,810	$12,834	$13,178
Euro	$7,758	$4,667	$5,863
Japanese yen	$5,319	$4,236	$4,344
Pound sterling	$2,612	$2,242	$2,479
Swiss franc	$937	$880	$906
Canadian dollar	$594	$647	$605
Swedish krona	$419	$459	$451
Other	$2,674	$2,723	$2,380
	By Maturity[3]		
Up to one year	$15,791	$12,140	$13,178
Between one and five years	$1,624	$1,539	$1,623
Over five years	$592	$666	$693
Total contracts	$18,011	$14,344	$15,494

[1] All figures are adjusted for double-counting. Notional amounts outstanding have been adjusted by halving positions vis-à-vis other reporting dealers.

[2] Counting both currency sides of every foreign exchange transaction means that the currency breakdown sums to 200% of the aggregate.

[3] Residual maturity.

Source: Press release dated November 13, 2000 (www.bis.org). Reprinted with permission of the Bank for International Settlements.

of its contracts, and this in turn would cause a second party to default, and so on. Some fear that such a domino effect would result in the collapse of our entire financial system and economy!

These fears are largely unfounded for several reasons.

1. Stating the market in terms of notional principal overstates the size of the market. The Government Accounting Office (GAO) has estimated that the present value of expected payments on existing swaps is only 1% of the notional principal amounts. The Bank for International Settlements (BIS) estimates that the estimated gross market value of all global OTC interest rate derivatives is only 2.2% of the reported notional amounts, and for OTC foreign exchange derivatives, its only 4.6%. These figures exclude netting (see item 2) and other risk-reducing arrangements. It is only this present value of expected swap cash flows that is at risk, since the notional principal is never exchanged. To illustrate, contrast an ordinary bond issue in the amount of $40 million (a fixed-income debt instrument) to a swap with a notional principal of $40 million. The principal amount of a bond is initially exchanged from the lenders to the borrower. At this time, the amount exposed to default risk is actually $40 million, which equals the present value of the remaining cash flows that the borrower is contractually obliged to repay the investors. If interest rates remain unchanged, and if no principal is ever retired, the amount exposed to default risk remains at $40 million. But now, consider the plain vanilla interest rate swap example we discussed just a bit earlier, in which I agreed to pay you 8% of $40 million each year and you agreed to pay me one-year LIBOR times $40 million. The $40 million notional principal is never exchanged. If LIBOR is expected to remain at 8% for the entire term of the swap, the present value of the expected swap cash flows is zero! And if one of the parties defaulted at any time that LIBOR was expected to stay at 8%, there would be virtually no economic impact caused by the default.

2. Only one party to a given swap has the incentive to default at any time. This party is the one that *expects* to make most of the remaining swap payments. Thus, a party to a swap may actually go bankrupt because of events external to the swap contract. But if this party expects its swap cash flows to be positive, it will not default on its swap obligations. Furthermore, most, if not all, swaps that originate now have bilateral netting agreements. This arrangement means that if a party defaults on one swap, then all other swaps to which it is a party and which have a positive value (the defaulting party expects to be receiving future cash flows) are also closed out. This prevents a party from defaulting only on the swaps on which it is losing money.

3. Many swaps are marked to market daily, so that there is never a large buildup of unrealized losses that would provide an incentive to default.

4. The less creditworthy parties to many swaps are required to post collateral.

1.3 Options

A **call** option is a contract that gives the owner the right, but not the obligation, to buy an underlying asset, at a fixed price, on (or before) a specific day. The call seller (usually known as the "writer") is obligated to deliver the underlying asset. Thus, options separate rights (option owners have rights) from obligations (option writers have the obligation to respond if the owner exercises his or her option).

The fixed price of an option is called the *strike price*, or the *exercise price*, and is symbolically denoted with the letter K. When an option can be exercised only on a specific day, it is called a European option. If it can be exercised at any time up to and including the expiration day, then the option is American. Options have value for the owner. At worst, the owner of a worthless option can throw it away, since he or she is not obligated to do anything with it.

A **put** option is a contract that gives the owner the right but not the obligation to *sell* the underlying asset at the strike price. The put seller (writer) is obligated to purchase the underlying asset, and pay the strike price for it. But the put seller will be forced to buy the underlying asset if and only if the put owner exercises the option.

Whereas forwards, futures (which are just portfolios of forwards), and swaps (which are also just portfolios of forwards) provide profit diagrams that are just 45° straight lines, option profit diagrams consist of kinked, piecewise linear portions. If there are kinks, they will always exist at the strike prices. Before we illustrate the profit diagrams for call options, let's define a set of new terms: in, out of, and at the money. If S denotes the price of the underlying asset, and K denotes the strike price, then a call is:

In the money if $S > K$

Out of the money if $S < K$

At the money if $S \sim K$ (i.e., S is close to K)

Deep in (out of) the money if $S >> K$ ($S << K$)

The payoff diagram of an option illustrates the cash flows that occur on the expiration day of the option. Figure 1.4 is the payoff diagram for the owner of a call option. If the call finishes out of the money, then it has expired worthless, the call owner will not exercise her option, and there is no expiration day cash flow. If, on the other hand, the option finishes in the money, the option owner will exercise her option and buy the underlying asset for $K. The asset itself is worth $$S_T$. So, the option owner is purchasing something for less than it is worth. Because the call is in the money at expiration, $S_T > K$, and the expiration day payoff to the owner of the call is $S_T - K$.

To generate a profit diagram, we now only need to account for the initial requirement that the owner pay for that option. Figure 1.5 shows that the payoff diagram is just uniformly lowered by the amount the call buyer originally paid for the option. Denote this initial cost (it is frequently called the option "premium") as C_0.

Calls are one type of options. Puts are the other type.

Figure 1.6 is the payoff diagram for a long put position, and Figure 1.7 is the profit diagram for the long put. A long position refers to the owner of the put.

You have used options many times and probably did not even realize it. If you own a home, you have probably borrowed money, perhaps for 15 or 30 years, to make the purchase. You also know that if you sell your house, or if interest rates drop sharply, you can pay your loan off early. This is called a *prepayment option.* A floating-rate mortgage, which has limits on how much the interest rate can rise or fall each year, is another example of an option; these limits are caps and floors, which are interest rate options. You have entered into an option agreement if you ever rented furniture or a car, with an option (a call) to buy. Perhaps you have purchased

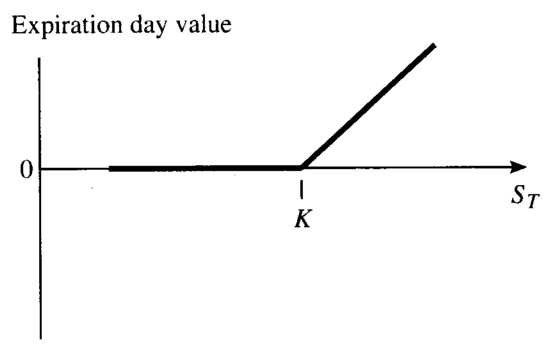

Figure 1.4 Payoff (at expiration) diagram for a long call.

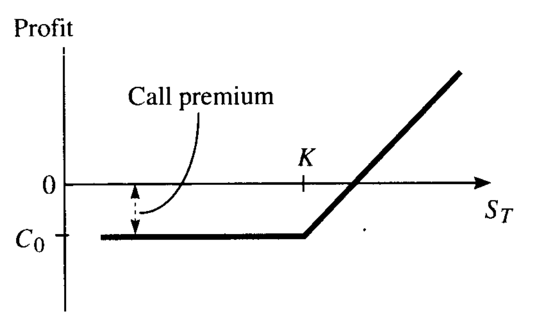

Figure 1.5 Profit diagram for a long call obtained by lowering the payoff diagram (Figure 1.4) by the call premium.

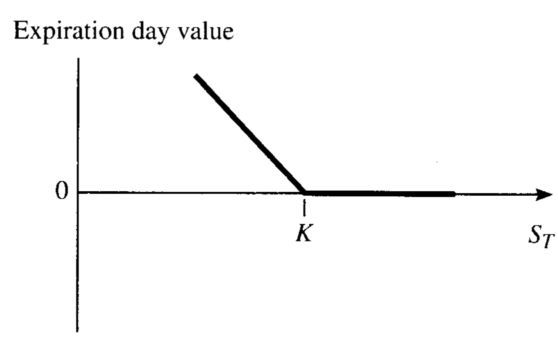

Figure 1.6 Payoff diagram for a long put.

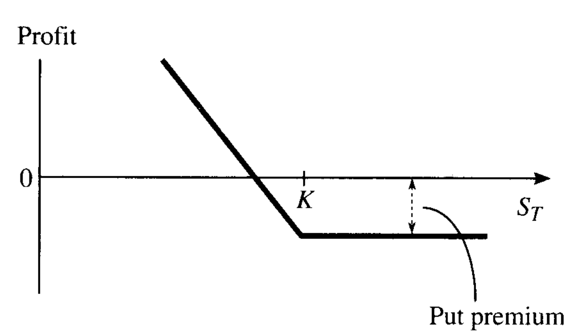

Figure 1.7 Profit diagram for a long put, obtained by lowering the payoff diagram (Figure 1.6) by the put premium.

options to get preferred seating at concerts and/or shows. When you purchase goods covered by a warranty, you have the option (a put) of returning the items within a specified period if you are unsatisfied in any way.

Options are often called "contingent claims" because their payoffs are contingent on some event. For example, option payoffs are contingent on whether they finish in the money or out of the money.

Options trading has a long history.[11] The concept of an option existed in ancient Greece and Rome. Options were used by speculators in the tulip craze of seventeenth-century Holland.[12] Unfortunately, there was no mechanism to guarantee the performance of the options' terms, and when the tulip craze collapsed in 1636, many speculators were wiped out. In particular, put writers would not take delivery of the tulip bulbs and refused to pay the high prices they had originally agreed to pay. Puts and calls, mostly on agricultural commodities, were traded in the nineteenth century in England and in the United States. Options on shares of common stock were available in the United States on the OTC market only until April 1973. The terms of these OTC options were negotiated by the buyer and seller, with a member of the Put and Call Brokers and Dealers Association serving as an intermediary or as one party to the contract. Thus an investor could buy a call option on 250 shares of AAA Corp. at a specified price of $26.125 per share, with an expiration date of the next April 30. Liquidity was lacking for these "custom-made" contracts, and each party had reason to be concerned about the other's ability to conform to the terms of the contract.

After five years of development that cost $2.5 million, the Chicago Board Options Exchange (CBOE) opened on April 26, 1973, in an old lunchroom at the Chicago Board of Trade.

Initially, 16 standardized call option contracts on individual stocks were listed on the exchange, and first-day volume totaled 911 contracts. Each founding member of the CBOE paid $10,000 for a seat (in 2001, the price of a seat was around $370,000). Put options began trading on the CBOE and four other options exchanges on June 3, 1977. On March 11, 1983, the first index option began trading on the CBOE; it was originally called the CBOE 100 Index, and later renamed the S&P 100 Index, or OEX (its ticker symbol). Options on futures contracts, which were temporarily banned by Congress in 1978,[13] were reintroduced to the marketplace on October 1, 1982. The first contracts were options on Treasury bond futures contracts (4080 T-bond futures options traded that day) on the CBOT, and sugar futures options on the New York Coffee, Sugar, and Cocoa Exchange.

The Bank for International Settlements estimated that at the end of 2000, the notional principal of the assets underlying outstanding exchange-traded options was $5.9 trillion. Actual trading activity of these options came to $66 trillion (notional principal) in 2000. Burghardt (2001) reports that in 2000, 156.9 million option contracts traded in the United States, and another 421.2 million contracts traded outside the country; these figures do *not* include options on individual stocks. Another 970 million option contracts on individual equities traded globally in 2000. A total of 578 million option contracts actually traded in 2000. The market for options that are created by securities dealers in the OTC market is even large: the BIS reported that the notional principal of the OTC options market was $13.2 trillion at the end of June 2000.

1.4 Why Is It Important to Learn About Derivatives?

One obvious motivation (at least to the authors) for learning about derivatives is the sheer size of the market. It is difficult to justify ignorance about a market that, as of the end of 1999, was estimated to be $88 trillion.

Certainly, any student who hopes to work in the financial marketplace should have some knowledge about derivatives and how she might be able to use these products in her job. These contracts can be used to control risk exposures or to speculate on future price movements. Firms, institutions, and governments can all use derivatives to manage risk and/or to increase profitability. The prices of many commodities, interest rates, currencies, and equities have become increasingly volatile at times in the recent past. Price volatility can result in financial hardship, even bankruptcy, for some firms. Derivatives allow the user to transfer these risks to other parties who are willing and able to accept them. Everyone who works in the world of finance must understand how derivatives work.

Derivatives can have a tremendous impact on spot markets, and this is yet another reason for reading this book. Hardly a week passes that financial commentators in the media do not attribute the stock market's price movements to **program trading**. A program trade is an order, usually placed by a large institution, to buy or sell large amounts of stock of many different companies. Put another way, it is the trade of a large portfolio of stocks as a group. Many program trades also involve options and/or futures. Indeed, many program trades are performed to arbitrage small price discrepancies that exist between the spot, options, and/or futures markets. The actions of derivatives traders affects stock prices, intrest rates and the prices of foreign currencies.

Options and futures trading has been blamed for almost every large stock market move since 1987, including the stock market crash of October 19, 1987, when the Dow Jones Industrial Average fell an unprecedented 508 points, down 22.6% on one day. If derivatives trading exists for a given commodity, then at times, that trading will almost certainly affect the spot price of that commodity. Thus options and futures trading has an impact on the lives of all consumers and suppliers of products and services, and on the experiences of all investors.

Last, but hardly least, you should understand futures and options to be able to protect yourself. Only if you understand derivatives can you decide intelligently whether you should ever use them. You will also be better able to avoid being defrauded or cheated.

Many investors lost huge amounts of money in the stock market crash of October 1987 because they listened to stockbrokers who advised that a particular strategy—writing naked puts—was a low risk route to profits.[14] Others have lost their life savings by speculating in futures, after being fast-talked into futures trading by high-pressure brokers, or by doing business with unscrupulous members of the futures industry. Individuals are not alone. Very large corporations (e.g., Procter & Gamble), financial institutions (e.g., Barings), and governments (e.g., Orange County, California) lost billions of dollars by using derivatives. By studying the material in this book, you will be better educated about such risks. You will also be better able to gauge the knowledge of any derivatives salespeople with whom you might do business.

Derivatives are not mere financial curiosities—the explosive growth in their number and usage is a direct result of their value in managing risks and returns. **Financial engineers** use derivative securities to manage risk and exploit opportunities to enhance returns. They create derivatives of different types that possess payoff patterns that meet investment or risk management needs, just as "typical" engineers create structures (manufacturing plants, bridges, roads, circuit boards) that meet the needs of the user.

In recent years, managing traditional price risks has mushroomed into managing the total risks of the firm. Financial engineers now have their own professional group, The International Association of Financial Engineers (www.iafe.org/). Risk managers also have a professional group, known as GARP, the Global Association of Risk Professionals (www.garp.com).

1.5 SUMMARY

In today's increasingly complex world of finance, both investors and financial managers must be aware of options and futures. They are important as speculative vehicles, and as risk management tools. This book takes the view that it is important for users of options and futures to know the principles of valuation and to learn how they can be used to manage price risk.

Speculators use derivatives (usually futures and options) to attempt to profit from expected price changes. Because derivatives are usually highly levered assets, huge profits can be realized if a speculator's prediction of the direction and amount of price change is correct. Consistent with the central concept that greater expected returns exist only where greater risks exist, it must be noted that many speculators have lost huge sums of money by being wrong about future price movements. Still, derivatives allow investors to establish, at low cost, return distributions that match up with their levels of risk aversion.

Hedgers use these contracts to control the risks they face regarding price changes. Hedgers include individuals, corporations, financial institutions, traders, and other entities facing the possibility that a price change will reduce the value of their net worth. Under ideal conditions, hedgers might be able to reduce the price risks they face to almost zero. Other hedgers might prefer the purchase of insurance, using options, rather than shrinking the range of possible future outcomes. Derivatives facilitate hedging and insuring. To successfully use these contracts, the basics of valuation and the principles of how derivatives can be used to manage risk must both be thoroughly understood, and this text attempts to promote such understanding.

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Notes

[1]*Futures Industry* lists 58 futures and options exchanges in existence in 26 different countries in 2000 (Burghardt, 2001, pp. 40–41).

[2]The notional principal is just the amount of money that is used to determine the cash flows of a swap. When the principal amount is notional, it is not actually exchanged; it simply represents the amount to which a formula is applied so that periodic cash flows can be computed.

[3]A standardized contract has terms that are identical to those of all other contracts. In other words, it specifies an exact amount of a good, the quality of the good, where and when it is to be delivered, and so on. All the contract's terms are stated, except the price. Thus, every gold futures contract that calls for delivery in a given month is identical to every other gold futures contract for that month. Standardization of futures contracts contributes to their liquidity.

[4]Much of the following history of forward transacting and futures trading is drawn from the Chicago Board of Trade's *Commodity Trading Manual*, (1985).

[5]Actually, foreign exchange futures began trading on the International Commercial Exchange (ICE), which was part of the New York Produce Exchange, in 1970. This venture failed after the IMM commenced operations, perhaps because of high initial margin requirements at the ICE, and perhaps because the ICE failed to adequately promote and market itself.

[6]A GNMA certificate is a security that is backed by a pool of mortgages. Investors in GNMAs receive the interest and principal that are paid by the homeowners, less some fees. The U.S. government guarantees these payments of interest and principal. The values of GNMAs rise as interest rates fall and decline as rates rise.

Volume in GNMA futures rose sharply in the early years of trading, reaching more than 9000 contracts per day in 1980. However, the contract's terms were complex, and other interest rate futures contracts became much more popular. Trading volume shrunk and in January 1988, the GNMA futures contract (in its original form, at least) ceased to exist. See Johnston and McConnell (1989) for an analysis on what contributed to the contract's failure.

[7]By "cash-settled," we mean that no delivery of the underlying good ever takes place. At delivery, all profits and losses are received and paid in cash.

[8]See note 2 in this chapter.

[9]LIBOR is the acronym for the London interbank offer rate. U.S. dollar LIBOR is the rate at which major London banks lend Eurodollar deposits. There is a LIBOR for each different maturity and for different currencies. Thus, we can speak of 3-month U.S. dollar LIBOR, and 6-month Japanese yen (JPY) LIBOR. "Eurodollars" is the term used for dollars that are borrowed and lent outside of the United States. Any dollars deposited in *any* bank located outside the United States (even if that bank is in Asia) are called Eurodollars. A more general term is **Eurocurrency**, which refers to any currency that is deposited in a bank located outside its country of origin (e.g., Euroyen are yen that are deposited in any bank outside of Japan).

[10]Das (1994, Chap. 1) provides a history of the swap markets.

[11]See Chapter 3 of Gastineau (1988), and Chapter 1 in *Options: Essential Concepts and Trading Strategies*, edited by the Options Institute (1990) for more details on the history of options.

[12]See Garber (1989) for details on the tulip bulb speculative bubble.

[13]In the 1970s, futures options were not traded on exchanges, but were instead often promoted by "boiler room" sales people, who pressured naive investors into buying the contracts. Furthermore, there was frequently no seller of the futures option, and by the time an investor tried to sell his option, the salesman and his operation had disappeared. Later, having introduced a stringent set of regulations and disclosure requirements, the Commodity Futures Trading Commission was able to persuade Congress to rescind the ban. Option contracts had been temporarily banned in England for several periods, beginning in 1733, when Bernard's Act was passed, until the 1950s.

[14]A front-page article in the December 2, 1987, issue of *The Wall Street Journal* recounts some of these horror stories.

PROBLEMS

1.1 On July 10, party A buys a forward contract from party B (B is the seller of the contract). The forward price is \$38, and delivery is on October 10. One month prior to delivery, the forward price for new contracts that call for the delivery of the underlying asset on October 10 is \$35. Which party is more likely to default on the contract on September 10?

1.2 Describe how OTC derivatives differ from derivatives traded on public exchanges.

1.3 "Both futures and swaps are equivalent to portfolios of forward contracts." Explain this statement.

1.4 If the price of the underlying asset rises, explain why being long a futures contract would likely be preferred to having a long forward position in that underlying asset, all else equal.

1.5 Explain why notional principal overstates the size of the derivatives market.

1.6 An arbitrageur

- **a.** tries to profit by trading mispriced assets
- **b.** tries to manage risk exposure via hedging
- **c.** tries to manage risk exposure via insurance

d. tries to profit by buying low today and selling high in the future

e. has a view on where the market is heading and tries to profit on those beliefs

1.7 Which of the following statements is true?

a. Futures have more default risk than forwards because the clearinghouse will protect traders of forward contracts.

b. Futures have more default risk than forwards because forward contracts are settled daily (marked to market daily).

c. Futures have more default risk than forwards because forwards are custom-made contracts.

d. Forwards have more default risk than futures because the clearinghouse will protect traders of futures contracts.

e. Forwards have more default risk than futures because futures are custom-made contracts.

CHAPTER 2

Risk and Risk Management

Learning about risk and risk management is an important key to understanding the modern financial landscape. You should be cognizant of the risks and opportunities you face as an investor, as an employee or owner of a business, and as a citizen.

This book deals with the nature of price risk and how you can use derivatives to manage it. A derivative is a contract whose value is derived from the price of some underlying asset. Chapter 1 presented an overview of four well-known derivative contracts that are used to manage risk: forwards, futures, swaps, and options. At the end of Chapter 1, we called your attention to two new professions, financial engineering and risk management.

Financial engineers and risk managers do not just manage a single price risk. Instead, they concern themselves with managing the total risk exposure faced by the firm, known as **enterprise risk exposure**. Managing this total enterprise risk is commonly known as **enterprise risk management**, or **ERM**. **Value at risk (VaR)** is a concept widely used in enterprise risk management. You will learn about VaR in chapter 20, but you will be introduced to some intricate details and uses of individual derivative securities in Chapters 3 through 19.[1]

The purpose of this chapter is to introduce you to the concepts of risk and risk management. As a starting point, recall the basic economic fact that firms utilize inputs to create outputs. If the revenues from the outputs sold are greater than all costs, then profits are earned. Output prices are one element of revenue, and input prices are an important component of total costs. Most firms are not in the business of speculating on the prices of inputs and outputs. Firms prefer knowing what prices will be in the future, so that they will have less uncertainty about the level of their profits. Consider the following simple income statement:

Revenues = Price of output times quantity sold
−Expenses = Prices of inputs times number of inputs required
Profits

Profit uncertainty is greatly reduced if a firm knows what the prices of its inputs will be and if it can have a guaranteed selling price of its output. Risk cannot be totally avoided, since the firm never knows exactly how much of its output it will be able to sell,[2] nor can it totally control the total cost of all of its inputs (e.g., labor expense, R&D expense, input spoilage, or the expense of raw materials used in undeliverable output). Many risks just cannot be managed. Unusual weather, for example, can affect the amount of goods or services that a firm sells, have an impact on its costs, and even cause catastrophic losses that destroy production capabilities.[3] Another example of a risk that cannot be controlled is competition. If another firm enters a market, existing firms will likely be forced to accept lower selling prices and a lower quantity of goods sold.

If you have had previous finance courses, you have been taught a great deal about risk. You have likely learned the concepts such as default risk, systematic (market) and unsystematic risk, variance, covariance, liquidity risk, and business risk vs financial risk. In this chapter, the following topics will be discussed:

What is risk?

How can risk be measured?

How can risk exposure be determined?

Should firms manage the financial price risks to which they are exposed?

It is certainly important to learn about risk management and derivatives. As you saw in Chapter 1, the use of derivatives has exploded, and by one measure, the size of the market is now over $88 trillion! Improper use of derivatives led to several well-publicized debacles in the 1990s. For example, you may have read about Metalgesellschaft, Procter & Gamble, or Long-Term Capital Management. Some firms and governments have lost billions of dollars through inappropriate trading and the lack of suitable internal controls. Notable recent cases that can be traced to the actions of one individual and the lack of oversight include Orange County (1994), Baring's Bank (1995), Daiwa Bank (1995), and Sumitomo Corp. (1996). A description of the procedures and technologies that can be used to deter inappropriate derivatives usage and subsequent losses is beyond the scope of this textbook. You should well note, however, that implementing and maintaining an effective enterprise risk management strategy is an important, burgeoning area.

Today, corporations, institutions, governments, and individuals routinely use derivatives to manage price risk exposure efficiently and effectively. According to Bodnar, Hayt, and Marston (1998), 83% of large nonfinancial U.S. firms use derivatives. Indeed, failing to use derivatives to deal with price risk is, in itself, dangerous, because it can be argued that firms that fail to manage price risk are simply speculating on prices.

2.1 WHAT IS RISK?

Risk is synonymous with uncertainty. Risk arises because the future is unknown. For example, default risk represents the future possibility that a party will default on a contractual obligation. Regulatory risk refers to the possible future imposition of rules, laws, or regulations that will impede doing business as it is currently done. Somewhat related is accounting risk, which would alter the way in which transactions are accounted for and reported on financial statements. Legal risk is the risk that a contract's terms will prove to be unenforceable, possibly because the instrument was poorly written. Price risk is one component of business risk, and it arises because the prices of goods will fluctuate in unpredictable ways in the future. Two common sources of price risk are foreign exchange rates and interest rates.[4]

What is important to understand is that the future is unpredictable, and the possibility of unexpected defaults in the future, as well as any regulations and price changes, all impact a business and affect future profits and values. Individuals dislike risk. Because they are often willing to pay a price to reduce uncertainty, they are risk-averse. The owners of financial securities, investors, are also risk-averse. Derivative contracts can be used to manage risk. Risk management can take the form of reducing (or even eliminating) price uncertainty via hedging transactions, or purchasing insurance using options to protect against unfavorable future outcomes while allowing participation in future favorable price changes.

Hedging and insuring do not make risk disappear. Instead, they transfer the price risk to those that are more willing and (hopefully) better prepared to accept them. Indeed, the chief economic benefit of derivative contracts is that they provide inexpensive conduits to transfer risk. Buying insurance on your house does not eliminate the risk that your house will burn down. Instead, the insurance contract allocates this risk to an insurance company that is both willing (because it charges you an insurance premium that should be sufficiently high to allow it to earn a profit) and better prepared (because it diversifies its risk exposures and is well capitalized) to deal with the event insured against, should it occur. Similarly, a firm that manages its exposure to the risk that interest rates will rise does not make the threat or the impact of rising interest rates disappear. The consequence of the higher interest rates (a decline in firm value or profits) is still realized. But because it has used derivatives, the firm has effectively transferred the impact to another party.

2.2 How Is Risk Measured?

This section contains a brief review of the elements of basic probability concepts that should be familiar from statistics and/or finance courses. In this section, we present the building blocks to help you understand other measures of risk that are often incorporated into enterprise risk management. That is, once you understand the basics of measuring risk, you can more readily refine your measure to include, for example, the likelihood of significant price changes.

2.2.1 Random Variables, Probability Distributions, and Variance

Future outcomes are random variables. An individual, firm, or institution facing a random variable is facing risk. In finance, the variance of a random variable is the most frequently used measure of risk.

Think of a random variable as a list of possible outcomes. We do not know which of the outcomes will be realized. If probabilities are attached to this list of possible outcomes, we then have a probability distribution of the random variable. Risk management alters the probability distribution of important random variables such as profits, rates of return, and stockholder's wealth.

Probability distributions may be discrete or continuous. A discrete distribution has a finite list of outcomes, each with its own probability. For example, suppose that the rate of return that will be earned on a common stock investment is a random variable denoted $\tilde{r}$.[5] Then this is a discrete probability distribution:

Outcome	Probability
−0.12	0.10
0	0.15
0.10	0.35
0.20	0.25
0.30	0.15

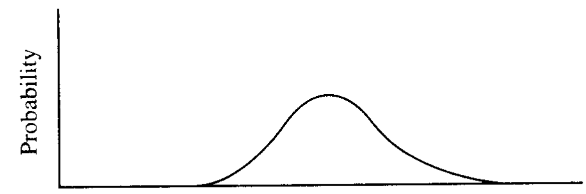

Figure 2.1 A continuous probability distribution.

Discrete distributions like this are unrealistic representations of the rates of return on financial assets. Nonetheless, many variables described in this text will be modeled as discrete distributions because they are more easily mathematically handled than continuous probability distributions. A continuous probability distribution has an uncountable number of possible outcomes. For example, Figure 2.1 is a depiction of a continuous probability distribution.

Although rates of return of risky assets are more likely to be continuous random variables, many future prices that we deal with are discrete random variables. These include the prices of commodities such as gold, oil, and wheat, as well as interest rates and the prices of foreign currencies. These random variables are discrete because they are expressed in some minimum unit of account, such as a penny. Other prices, such as stock prices, are constrained by trading rules to be discrete, formerly multiples of 1/8 or 1/16 and now in pennies.

Some assets (such as a pure discount Treasury bill) are riskless, so the rate of return over the asset's life is known with probability 1. Today's prices (spot prices) are known, fixed, and certain. Moreover, there may even be fixed prices quoted today for the delivery of goods in the future; these are called forward prices. But the actual spot price that will exist in the future is a random variable. When we consider a collection of random variables, such as the time series of daily prices during the next year, it is said that we are dealing with a stochastic process.

A probability distribution is characterized by certain measures that are known statistically as **moments**. Given a set of moments (i.e., central tendency, dispersion, skewness), we can create an entire probability distribution. For example, since a normal distribution exhibits no skewness, such a distribution is entirely described once we know its mean and variance.

The first moment, the expected value, is a measure of central tendency. It has the characteristic that if an infinite number of returns were independently drawn from an unchanging probability distribution, then the expected return would be the average value of the random variable.

The expected return of a random variable is a probability-weighted average of the possible outcomes. It is computed by multiplying each outcome by its probability and then adding all the resulting products. For example, the expected return of the discrete random variable just presented (the expected return of the common stock) is computed as follows:

1	**2**	**3**
Outcome	**Probability**	**Outcome × Probability: col. 1 × col. 2**
−0.12	0.10	−0.012
0	0.15	0
0.10	0.35	0.035

0.20	0.25	0.05
0.30	0.15	0.045
	Σ = expected return = 0.118 = 11.8%	

It is interesting to note that the expected return is not equal to any of the discrete outcomes that are possible. If $\tilde{r}$ is a discrete random variable, and there are n possible outcomes, then the expected return $E(\tilde{r})$ is

$$E(\tilde{r}) = r_1 \Pr(\tilde{r} = r_1) + r_2 \Pr(\tilde{r} = r_2) + \cdots + r_n \Pr(\tilde{r} = r_n)$$

For this example,

$$E(\tilde{r}) = (-0.12 \times 0.10) + (0 \times 0.15) + (0.10 \times 0.35) + (0.20 \times 0.25) + (0.30 \times 0.15) = 0.118$$

The short-cut notation for the expected return is

$$E(\tilde{r}) = \sum_{i=1}^{n} r_i \Pr(\tilde{r} = r_i)$$

Another widely used measure of central tendency is the median. The median occurs at the value at which there is a 50% chance that the outcome will be below it and a 50% chance that the outcome will lie above it. In the foregoing example, the median lies between 0 and 0.10. This is found by ordering the outcomes from lowest to highest (as they already are), and cumulating the probabilities as we move from the lowest to the highest value, as follows:

Outcome	**Probability**	**Cumulative Probability**
–0.12	0.10	0.10
0	0.15	0.25
0.10	0.35	0.60
0.20	0.25	0.85
0.30	0.15	1.00

To find the median, we must find the value at which the cumulative probability equals 0.50. This occurs at an outcome greater than 0 (where the cumulative probability is 0.25) and less than 0.10 (where the cumulative probability is 0.60).

In addition to measures of central tendency, we are interested in measures of dispersion. The greater the dispersion, the less certain we are about what the outcome will be, and the greater the risk.

The simplest measure of dispersion is the range of outcomes. The range is just the difference between the highest and lowest outcomes. In our example, the range is 0.42 [0.30 – (– 0.12) = 0.42].

The range is not an informative measure of dispersion. The range is the same regardless of whether there are just two possible outcomes (0.30 and – 0.12) and the probability of each is 0.50, or many different outcomes and the probability of one of the extreme outcomes is very high and the other extreme outcome is very low. Alternatively, there may be many different outcomes, with the probabilities of each of the two extreme outcomes being very low (say, 0.00001).

The second moment of a probability distribution, and a measure of a random variable's dispersion, is its variance. The definition of variance is "the expected value of the squared deviation from the expected value." While this sounds like something Groucho Marx might say, it is actually quite simple. The calculation of the variance of a random variable can be illustrated as follows.

1	2	3	4	5
Outcome	**Probability**	**Outcome × Probability: col. 1 × col. 2**	**Squared Deviation from the Expected Value**	**Probability × Squared Deviation: col. 4 × col. 2**
−0.12	0.10	−0.012	$(-0.12-0.118)^2=0.056644$	0.0056644
0	0.15	0	$(0-0.118)^2=0.013924$	0.0020886
0.10	0.35	0.035	$(0.10-0.118)^2=0.000324$	0.0001134
0.20	0.25	0.05	$(0.20-0.118)^2=0.006724$	0.0016810
0.30	0.15	0.045	$(0.30-0.118)^2=0.033124$	0.0049686
	Σ = expected return = 0.118 = 11.8%			Σ = variance = 0.0145160

We have already discussed the first three columns. Column 4 takes the deviation of each outcome from the expected value and squares it. For example, the first outcome is −0.12. The difference between that outcome and the expected value of 0.118 is 0.238 (−0.12−0.118). The square of 0.238 is then 0.056644. This process of finding the squared difference between an outcome and the expected value is repeated for each outcome in column 4.

Whenever you see the word "expected" before another word or set of words, you should immediately think in terms of probability weighting. This is what is done in column 5. Because variance is the expected value of the squared deviation from the expected value, we must attach probability weight to the squared deviations from the expected value. For example, the first squared deviation from the expected value is 0.056644. If we multiply that number by the probability of its occurrence, 0.10, we get the number on the first line of column 5, namely: 0.0056644.

Finally, to complete the computation of variance, add the probability-weighted squared deviations from the expected value, and we find that the variance is 0.0145160.

The units of this return variance is "percentage-squared." The main drawback of variance is that it is difficult to understand the meaning of percent-squared. Thus, standard deviations are often utilized. Standard deviation is the square root of variance. The standard deviation of returns in our example is $\sqrt{0.0145160} = 0.12048 = 12.048\%$.

We can use the following formula to summarize the computational procedures of all these parameters:

$$\mathrm{Var}(\tilde{r}) = \sigma^2(\tilde{r}) = E[r_i - E(\tilde{r})]^2 = \sum_{i=1}^{n} [r_i - E(\tilde{r})]^2 \Pr(\tilde{r} = r_i)$$

Expanded, this becomes:

$$\mathrm{Var}(\tilde{r}) = [r_1 - E(\tilde{r})]^2 \Pr(\tilde{r} = r_1) + [r_2 - E(\tilde{r})]^2 \Pr(\tilde{r} = r_2) + \cdots + [r_n - E(\tilde{r})]^2 \Pr(\tilde{r} = r_n)$$

The standard deviation (SD) is the square root of the variance:

$$\mathrm{SD}(\tilde{r}) = \sigma(\tilde{r}) = \sqrt{\mathrm{var}(r)}$$

The greater the variance of a desired variable, such as a rate of return on an asset, the greater its risk. As an outcome becomes more certain, its variance and standard deviation become smaller. Another word that we use interchangeably with variance is volatility. Iomega is said to be a more volatile stock than IBM because the probability distribution from which its rates of return are drawn has a greater variance than IBM's. Because we assume that most individuals are risk-averse, we also assume that most people will pay a premium to reduce variance. Individuals and firms might wish to use derivatives to reduce the variance of the outcomes they face when dealing with prices. Figure 2.2 compares the variance of the profit on a transaction if the price risk is unhedged, to the variance of a transaction's profit if price risk is hedged.[6]

In this book, the standard deviation will often be used to measure risk. This is not to say that standard deviation is the *only* definition of risk. For many individuals and firms, other risk measures are more relevant. For example, many people regard risk as the probability of losing money (the probability of realizing a rate of return less than zero). More generally, they may regard risk as the probability that the rate of return or price is below some critical value a: $\Pr(\tilde{r} < a)$. Others might focus only on the worst possible outcome. The semivariance might be what yet others concentrate on, since only outcomes below the expected value are regarded as being undesirable. The semivariance of a probability distribution is its variance measured only over outcomes that are less than the expected value. If the probability distribution is symmetric, like the normal distribution, then the semivariance and the variance measure risk in identical ways.

The third moment of a probability distribution is its skewness, which arises when the probability distribution is asymmetric.[7] Rather than presenting a formula for skewness, we will illustrate it with a sketch. Figure 2.3a depicts positive skew, and Fig. 2.3b shows negative skew. If the random variable is something desired, such as the rate of return on an asset, then skewness will also be desired, and investors will be willing to pay for positive skew. The purchase of insurance creates a positively skewed outcome, since the downside is truncated at some minimum amount, while the upside still exists.

2.2.2 Measuring Risk Using Past Data

Is the price of oil any more risky than the price of gold? Are short-term interest rates more uncertain (i.e., more volatile) than long-term interest rates?

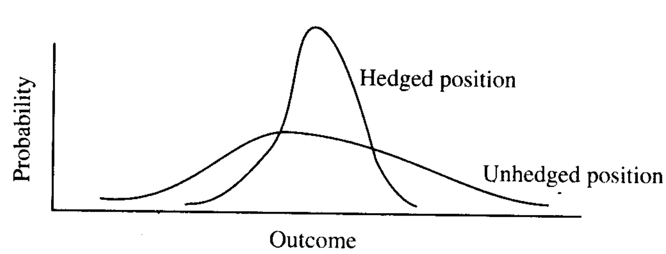

Figure 2.2 The probability distributions for an unhedged and a hedged transaction.

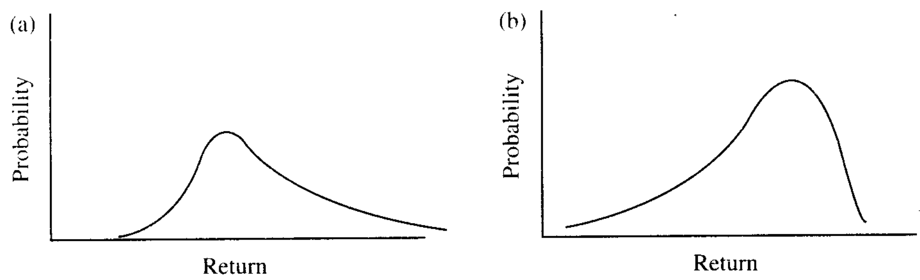

Figure 2.3 (*a*) Positively skewed random variable. (*b*) Negatively skewed random variable.

These questions cannot be precisely answered, although the finance discipline tries. In finance, we like to think that a future price, or a future rate of return, will be drawn out of a probability distribution. The distribution has a variance, and this parameter measures the risk, or uncertainty, regarding that price or rate of return. The greater the variance, the greater the risk. But this distribution is unobservable. Each of us might have a subjective idea of what this distribution is, but no one really knows. In addition, the distribution from which the outcome is drawn may be *time varying* (i.e., changing over time). For example, when interest rates are high and there is a great deal of risk in the stock market, the return on the market might be drawn from a distribution with an expected value of 15% and a standard deviation of 25%. When interest rates are low and stock market risk is low, the market return distribution might have an expected value of 10% and a standard deviation of 20%.

The field of statistics uses historical data to allow us to make inferences about the variance of the distribution from which future outcomes will be drawn. Assuming that past observations were independently drawn from the same (an identically distributed) probability distribution, we can use past data to estimate the mean and variance of the underlying distribution. By "independent" we mean that no outcome was affected by any previous outcomes and will not affect any future outcomes; the occurrence of any outcome does not affect the probability of past and future outcomes. By "identically distributed" we mean that the probability distribution from which the past observations were drawn has remained constant, and it will remain constant in the future. In other words, the expected value and variance have not changed over time.

Prices are *not* independent and identically distributed. The probability distribution from which IBM's historical stock prices were drawn almost certainly looked very different when shares were selling for $50 a piece and when they were selling for $150. It should be obvious, for example, that the means of the two price distributions were different. If today's price is $50, then the mean of the probability distribution from which tomorrow's price is drawn will be around $50. If today's price is $150, then the mean will be around $150. Also, daily prices are not independent. Tomorrow's stock price is very much dependent on today's stock price. Using past *price* data to estimate the variance of the probability distribution of next year's price (a random variable) is not advised.

However, changes in price and rates of return are much more likely to be independent and identically distributed. Thus, past data can be used to estimate the expected value and variance of a security's rate of return distribution. Past data on percentage changes in interest rates can be used to estimate the expected percentage change in interest rates, and the variance of the percentage change in interest rates.

If $\{r_1, r_2, \ldots, r_N\}$ represent N historical observations of the percentage change in some variable of interest, then the expected rate of return can be estimated using the sample mean $\bar{r}$:

$$\bar{r} = \frac{1}{N}\sum_{t=1}^{N} r_t$$

The sample variance is a good estimator of the variance of the distribution from which the percentage changes of the variable were drawn. The sample variance is denoted s^2, and it is computed by means of the following formula[8]:

$$s^2 = \frac{1}{N-1}\sum_{t=1}^{N}(r_t - \bar{r})^2$$

2.2.3 Identifying Risk Exposure

Firms will often hedge individual transactions that induce financial price risk into their business operations. For example, if Merck plans on borrowing €50 million (50 million euros) next year, it is exposed to the risk that euro interest rates will be higher when the company does raise those funds. The intended act of borrowing in the future is a planned individual transaction, and Merck can take steps to protect itself against that risk. Another example of a transaction occurs if Sony has just sold some electronic gear in the United States and will soon receive dollar-denominated receipts; Sony is then exposed to the risk that the value of those dollars, in terms of yen, will be lower in the future. Next year's interest rate for the euro currency, and the future ¥/$ exchange rate, are random variables that in each case will be drawn out of unique probability distributions.

A more advisable tactic is to analyze the firm's overall exposure to economic risks, that is, to measure the enterprise risk exposure. It is quite possible that at the same time one business unit is transacting in such a way that it is becoming exposed to the risk that the dollar price of yen will rise, another business unit is transacting so that it faces the risk that the yen will decline in value. Overall, the entire firm may be hedged. If each business unit separately hedges its exposure, the firm is just creating additional expenses for itself. Instead, a centralized risk management department can determine the firm's overall risk exposure and hedge only the residual risk that exists after such internal hedges have been identified. As another example, many diversified energy companies are internally hedged (partially) against fluctuations in the price of crude oil. If crude oil prices decline, the value of their oil production and reserves is lower. But if oil is also an input to the production of other goods the oil company manufactures, profit margins on these goods might widen if crude oil prices decline.

Besides being aware of all transactions, the overall firm has made and will make in the near future, risk managers must understand that the quantities of inputs the firm needs and outputs it sells may be a function of prices. Changing prices could affect planned capital expenditures, optimal transportation methods and costs, and purchasing, inventory, and distribution decisions. Finally, competitive pressures may also depend on prices, and these will create yet other impacts on profitability and value.

Thus, a firm should carefully assess the nature of its overall risk exposures. This is not an easy task. Even if it concludes that it is exposed, say, to the risk that interest rates will rise, the firm must also assess to what degree it is subjected to this risk. Will a 200-basis-point rise in interest rates cause the value of the firm's stock to decline (by 1%, 5%,...) or cause it to declare bankruptcy? In other words, the extent to which the firm is exposed to a price risk—that is, the set of consequences—is as important as merely determining that a risk exists.

A firm can use various approaches to determine its overall risk exposures. All these approaches try to estimate the relationship between changes in firm value or profits, and changes in prices. The first approach is to estimate previous historical cash flows. These will likely be quarterly figures, and they should be adjusted for any seasonal effects. The firm must decide which currency it should use when determining the amounts of these historical cash flows. Sometimes this is not obvious. For example, Shell Oil is a Dutch company that gets a substantial amount of its crude oil from British North Sea oil fields and sells much of its final product in the United States. Its common stock is listed and actively traded both in the Netherlands and in the United States; it is owned by investors in countries all over the world. Should it compute its cash flows in terms of Dutch guilders, U.S. dollars, or British pounds? The firm must decide the currency in which it wants to maximize profits and value.

Then, the firm can estimate the historical relationship between the changes in these cash flows and changes in the prices of the products to which they have risk exposure. To get a full understanding of the nature of its risk exposure, the firm should perform this analysis both on univariate and on several multivariate bases. For example, Shell may believe that it is exposed to the risk of changing short-term U.S. interest rates, to changing values of the \$/£ exchange rate (the symbol £ is used to denote the British pound), and to changing crude oil prices. It will first determine historical quarterly changes in its dollar-denominated cash flows (denoted ΔCF), and quarterly changes in 90-day Eurodollar rates (Δr), the \$/£ exchange rate (Δ£), and the dollar price per barrel of crude oil (Δoil). Shell could then estimate the following regression models:

1. Univariate regressions
   a. $\Delta CF_t = a + b(\Delta r_t) + \varepsilon_t$
   b. $\Delta CF_t = a + b(\Delta £_t) + \varepsilon_t$
   c. $\Delta CF_t = a + b(\Delta \text{oil}_t) + \varepsilon_t$
2. Multivariate regressions
   a. $\Delta CF_t = a + b(\Delta r_t) + c(\Delta £_t) + \varepsilon_t$
   b. $\Delta CF_t = a + b(\Delta r_t) + c(\Delta \text{oil}_t) + \varepsilon_t$
   c. $\Delta CF_t = a + b(\Delta £_t) + c(\Delta \text{oil}_t) + \varepsilon_t$
   d. $\Delta CF_t = a + b(\Delta r_t) + c(\Delta £_t) + d(\Delta \text{oil}_t) + \varepsilon_t$

In these models, the regression intercept a is a constant. The regression slope coefficients b, c, and d each estimate the sensitivity of changes in the firm's cash flows to changes in prices. If the firm is truly exposed to the risk that the \$/£ exchange rate will rise, then in models 1b, 2a, 2c, and 2d, the slope coefficients for the $\Delta £_t$ variable should be negative. A negative coefficient means that if the \$/£ exchange rate rises, cash flows decline.

There are several drawbacks to this approach, starting with its focus on what has happened in the past. Firms change their assets and liabilities, change their product lines, and change where they import from and export to over time. Competition changes over time. All these changes effectively result in time-varying risk exposures, which means that the slope coefficients are not constant. And of course, no matter how accurate the estimation of *past* risk exposure, there is no guarantee that a firm's exposure will remain constant in the future. Next, the firm must be sure that the cash flows it estimates by using past data do not include the impact of any risk management activities it may have undertaken earlier; this may be difficult to determine. Perhaps even more important, the firm must be sure that it is dealing with actual cash flows, not accounting variables, whose creation reflects a great deal of discretionary decision making. Finally, historical cash flows may fluctuate because of factors other than interest rates, exchange rates, and crude oil prices. But because these factors were prominent in quarters during which one or more of the three risk variables changed, the regression models will conclude that cash flows are dependent on those risk variables.

Another similar approach to identifying risk exposure merely substitutes the percentage changes in the firm's stock price (rates of return) for ΔCF in the foregoing regression models. In other words, the firm will determine how investors regard the relationship between equity value and changes in these prices. If investors perceive that the firm is exposed to the risk that the \$/£ exchange rate will increase, then when one is estimating a model in which the stock's rate of return is the dependent variable and the \$/£ exchange rate is the independent variable, the slope coefficient should be negative. While it is useful, this approach hinges on investors' abilities to identify the firm's risk exposures accurately and to incorporate them into security prices.[9]

Finally, if the firm is not confident about the results obtained by using historical data, it can estimate its own model of how its cash flows are determined and use Monte Carlo simulation techniques to simulate the results of the model. Two sets of variables must be estimated. First, the firm must identify how a change in a price will likely affect its cash flows. An increase in interest rates might slow the economy, leading to a reduced level of sales. Also, interest expense will rise. Additionally, creditors might pay off their payables more slowly, and some may even default. Will higher interest rates in this country have a secondary effect on exchange rates and foreign competition and foreign suppliers? All the direct and indirect cash flow impacts of the change in interest rates must be identified and quantified. The second variable that must be identified is the probability distribution from which future interest rates will be drawn. Is it a uniform distribution, with roughly equal probabilities that interest rates will be between 3 and 7%? Or, is it a normal distribution or skewed distribution?

Once the effect of changing prices has been estimated, and a probability distribution for the price hypothesized, Monte Carlo simulation has the researcher randomly draw many, perhaps 10,000 or more, interest rates from the distribution. For each interest rate drawn, there is a predicted impact on the firm's cash flows. Thus, the firm obtains a probability distribution of future cash flows that are dependent on its exposure to the price risk being investigated. The firm can then take steps to modify its exposure if it believes that doing so is beneficial. It can alter its operations, or use derivatives, to manage its risk exposure.

The Monte Carlo approach is good because it is forward looking, and because it disciplines a firm into analyzing the nature of its business risks. The impact of changing prices on *all* the firm's activities can be estimated by using Monte Carlo simulation. But, the Monte Carlo approach suffers from the drawback that the individual setting up the model will often impose personal biases, consciously or unconsciously, on the model. This may ensure that some particular results reflecting these biases will be obtained.

In this section, we have not described every method to measure and manage risk. New methods to measure risk are constantly being introduced. As a result, it is impossible to discuss them all. However, after reading this section, you should have an understanding of the nature and importance of measuring risk.

2.3 Should Firms Manage Risk?

Many observers would answer this question with a quick "Yes." After all, individuals are assumed to be risk-averse. This means that the firm's managers are risk-averse. Stockholders are risk-averse. And other stakeholders in the firm—employees, customers, suppliers, and so on—are risk-averse. So it would seem to make sense that taking steps to reduce risk would be an act desired by all concerned parties. A decline in the volatility of a firm's profits would appear to result in a lower cost of capital, a greater demand for the firm's shares, and a greater firm value.

On the other hand, some investors may prefer that the firm not hedge, so that they can capture all the profits, should some commodity price, interest rate, or currency price change in a forecasted direction. Suppose you were convinced that gold and oil prices were about to rise and decided to buy the shares of several gold producers and crude oil drilling firms. If you were right about the price increases, you would not be pleased to learn that these firms had hedged their production of gold and oil by selling gold and crude oil futures. What is the obligation of the firm to its stockholders in this case?

One of the most important results in all of finance is that when markets are perfect,[10] financial policy decisions such as the risk management decision do not affect firm value. One way of explaining this result is by asking a question: If stockholders can manage risk to fit their beliefs and level of risk aversion, why should they be willing to pay more for a firm that manages risk? The answer is that they shouldn't. If individuals can hedge, maybe firms should not.

It then follows that if corporate risk management is to increase the value of the firm, the reasons for the success of the venture must lie in market imperfections. In the following subsections, we briefly explain the factors that firms should consider before deciding to manage their exposure to financial price risk. The more applicable these reasons are, the more essential is a plan to manage risk.

2.3.1 Hedging Reduces the Expected Costs of Financial Distress

The value of the firm equals the present value of future expected cash flows that will be received by bondholders and stockholders. At each future date, there is some probability that the firm will default on its contractual debt obligations. In a perfect market, if the firm defaults, the creditors costlessly take control of the firm's assets with no disruptions. But in reality, if the firm defaults, there may be actual direct costs of bankruptcy that the firm will bear. These include additional legal and accounting expenses. In addition, the market may perceive an increasing bankruptcy probability long before the firm actually does declare itself insolvent. We call any additional costs that the firm bears before actual bankruptcy the *costs of financial distress*. In addition to added legal and accounting fees in these future states of financial distress (bankruptcy is the most extreme state of financial distress), there are other costs firms may face:

a. Customers want a viable firm to stand by its product warranties. If customers believe the firm may not survive in the future, they will not be willing to buy its product.

b. Suppliers may not be willing to extend trade credit. Instead, they may demand to be paid in full before providing the raw materials necessary to produce the firm's output.

c. Employees will, all else equal, demand to be paid a premium before they agree to work for a firm in financial distress.

d. Firm management will be distracted by financial distress. Their time will be spent in less productive activities because they will be dealing with tasks that have arisen as a result of the firm's deteriorating financial condition.

e. Investment opportunities may be passed over if the firm is denied access to the capital required to fund them.

f. Losses can be carried back for tax purposes. But once the tax-loss carrybacks have been exhausted, they must be carried forward. This practice decreases firm value because the time value of the tax benefits of negative taxable income makes them more valuable, the sooner they are used. If the firm fails to return to profitability in the future, the tax benefits may be lost forever.

Risk management actions that reduce the variance of the firm's cash flows will reduce the likelihood of its experiencing financial distress, and therefore will reduce expected financial distress costs. Suppose that the firm has $\$X$ of future fixed financial obligations, including interest payments, bond principal payments, and lease payments. Failure to meet any of these contractual obligations will result in bankruptcy. Investors have beliefs about what the firm's cash flow distributions will be in future years. The probability that its future cash flow will be below X is directly related to the firm's financial distress costs. As shown in Figure 2.4, hedging reduces the probability that its cash flow will be less than X. Therefore, the firm's expected costs of financial distress will be less and firm value will be greater if it has hedged itself than if it has not. A very effective risk management program might reduce to zero the probability that the firm's cash flows will be less than X.

2.3.2 Hedging Makes It More Likely That Future Attractive Investments Will Be Made by the Firm

Theoretical models have demonstrated that firms may forego positive net present value (NPV) projects when the bondholders will realize most of the investments' benefits, and this is most likely to happen when firms are in financial distress. Firms may even have incentives to accept negative NPV projects that are very high risk, since when the risky project proves to be very profitable, the stockholders will benefit. As an unrealistic but illustrative example, suppose that the firm has \$6 of

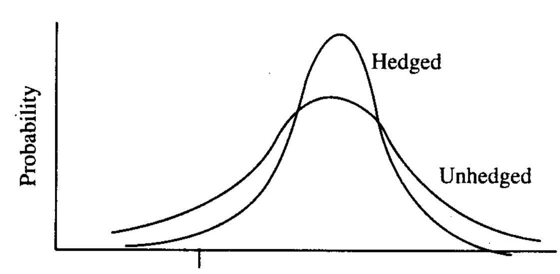

Figure 2.4 Probability distribution of the cash flows for a hedged and an unhedged firm. The firm's costs of financial distress are directly proportional to the probability that cash flow will be less than the firm's fixed financial obligation, which is $\$X$. Pr(unhedged cash flow) $<\$X$ is greater than Pr(hedged cash flow) $<\$X$. Thus, expected cash flows to bondholders and stockholders are greater for the hedged firm because of reduced expected costs of financial distress.

bond principal due to be repaid next year. Today, the firm has $3 to invest in one of two projects, which offer the following payoff distributions:

	Payoffs One Year Hence	
Probability	**Project A**	**Project B**
0.30	$5	$0
0.39	$5	$0
0.30	$5	$0
0.01	$5	$10

Even though project A is a positive net present value project at any discount rate less than 66.7%, and project B is almost certain to be a negative net present value at any discount rate, a firm whose investment decisions are made for the benefit of the stockholders will adopt project B. Why? Because by accepting project A, the firm would be forced to default no matter what the outcome; the bondholders would reap all the benefits of project A. At least if project B is accepted, there is a 1% chance that the stockholders will benefit, and will continue to control the firm's assets.

This unrealistic example illustrates the nature of the investing decision problem that arises when firms are in financial distress. A firm's stockholders will prefer that management "roll the dice," hoping for a favorable outcome from which they will profit. A firm in financial distress may invest in high risk, lower (and even negative) NPV projects. Bondholders recognize this before the fact and demand to be compensated for such risks. By hedging, these situations become less likely, and therefore the added interest costs of debt, and the possible failure to accept positive NPV projects, are averted.

Another model (Froot, Scharfstein, and Stein, 1993, 1994) that justifies risk management activities is based on the argument that internally generated funds are cheaper than any source of external capital. When future cash flows are unexpectedly low, firms may even be denied access to the capital markets (i.e., the cost of external capital then becomes infinite), and therefore fail to make new investments. By reducing the variance of future cash flows, risk management makes it less likely that the firm will be forced to try to raise costly external capital; that is, it increases the probability that the firm will always have sufficient internally generated cash flow to fund new investments.

2.3.3 It Is Less Costly for the Firm to Hedge Than for Individuals to Hedge

Trading commissions and collateral requirements for using derivatives will likely be less for firms than for individual investors. Indeed, many investors do not have access to many of the risk management tools (e.g., swaps) available to large firms. Many investors are unsophisticated and do not know how to hedge.

2.3.4 Firms May Have Better Information Than Individuals

It is unlikely that any firm, government body, financial institution, or individual can predict future interest rates or currency prices very well. However, when it comes to product prices, some firms may very well have information that is superior to individuals' information. For example, the Hershey Food Corporation probably has better information than most about the future supply of cocoa, and therefore should be skilled at managing this product price risk. Oil companies also have more accurate data about crude oil inventories, production, and demand than parties not involved in energy on a daily basis.

In addition, firms likely have a better conception than individual investors of what their risk exposures to all financial prices (including interest rates and foreign exchange) are. Certainly, they know the risks generated by individual transactions; investors have no access to information about individual firm transactions. For example, suppose a firm's managers know that debt will have to be issued 6 months hence. The firm can hedge against the risk that interest rates will be higher. Individuals will not be privy to the firm's future financing plans.

2.3.5 Nonsystematic Risks Should Be Hedged When Owners Are Not Well Diversified

The finance discipline often dichotomizes risk into a systematic portion and a nonsystematic portion. Investors are compensated (in the form of a higher expected rate of return) for bearing systematic risk. But because rational investors diversify, they are not compensated for bearing nonsystematic risk. For example, an investor can effectively hedge against a decline in the value of the dollar by investing in exporters who benefit from a weaker dollar, that is, those based in the United States. Portfolio diversification is certainly cheaper than having each firm hedge its own risk exposure (ignoring all the other reasons for hedging that are presented in this section). Each investor can use his or her own set of beliefs and degree of risk aversion to make the hedging decision.

However, if a firm is owned by a small group of investors, each of whom has a substantial amount of personal wealth in the form of that company's stock, it might be beneficial for that closely held firm to hedge its risk exposure, even though that risk is nonsystematic (diversifiable).

2.3.6 Hedging Can Increase Debt Capacity

If a firm's risk management activities increase the amount of debt it is able to issue, then it has increased the present value of its interest tax shields and increased firm value. Also, some lenders will lower the interest rate it charges a firm if it has hedged.

2.3.7 Risk Management Can Reduce Taxes

One effect of risk management on taxes has already been mentioned. By reducing the likelihood of future years in which negative taxable income is earned, hedging reduces the probability that a firm will be forced to carry tax losses forward. When tax losses are carried forward, their time value is lost.

But even beyond the lost time value, hedging increases average net income and cash flow because of the tax code. Items that create tax shields are lost if taxable income is too low. For example, interest expense and depreciation expense generate tax shields. The interest tax shield in a year equals $t \times \text{int}$, where t is the firm's marginal tax rate and int is its interest expense. The depreciation tax shield equals $t \times \text{dep}$, where dep is the firm's depreciation expense. By maintaining these tax shields, hedging increases firm value by both increasing average reported net income and actual average cash flow. This is illustrated in Table 2.1.

In Table 2.1, an unhedged firm faces a 50% chance that it will be profitable in any year, and a 50% chance that it will not be profitable. Profitability is measured by earnings before interest,

TABLE 2.1 How Expected Net Income and Expected Net Cash Flow Are Higher When the Firm Is Hedged than When It Is Not Hedged

	Unhedged Scenario		Hedged Scenario	
	Growth	**Recession**	**Growth**	**Recession**
Probability	50%	50%	50%	50%
Gross profit	300	0	150	150
− Depreciation expense	−80	−80	−80	−80
− Interest expense	−20	−20	−20	−20
Taxable income	200	−100	50	50
− Taxes (30%)	−60	0	−15	−15
Net income	140	−100	35	35
Net cash flow	220	−20	115	115
Expected net income	20		35	
Expected net cash flow	100		115	

depreciation, and taxes (EBIDT). If the firm is profitable, its EBIDT will equal \$300. If it is not profitable, its EBIDT will be zero. Profitability is determined by the state of the economy (recession or growth).

By hedging its operations against some price risk, such as energy costs, the firm will lock in its average EBIDT of \$150 [(\$300 + \$0)/2 = \$150].

Interest expense and depreciation expense are deducted from EBIDT to compute taxable income. Subtract the taxes due, 30% of taxable income, and we obtain net income. Cash flow in this simple world is defined to equal net income plus depreciation. The example in Table 2.1 shows that average net income and average cash flow are greater when the firm is hedged than when it is not. If the hedged firm's average cash flow is discounted at the same or lower discount rate,[11] then we can conclude that hedging increases the value of the firm. If the market applies the same or higher price–earnings multiple to the hedged firm's average earnings per share, then again we must conclude that hedging increases the value of the firm's common stock.

The source of these conclusions lies in the nature of the tax code. The example presented in Table 2.1 assumes no tax rebates. Note that if the firm was supplied with a \$30 tax rebate in the event that taxable income is –\$100 (this is the taxable income in the unhedged/recession scenario), expected net income and expected cash flow would be the same regardless of whether the firm is hedged. This would be the case if the tax loss could be carried back.

Another model exists to illustrate how hedging reduces taxes. It rests on the assumption that the tax schedule is convex. A convex tax schedule is one in which the average tax rate increases at an increasing rate as taxable income increases. This is illustrated in Figure 2.5.

The following tax table creates a convex tax schedule:

Taxable Income	Tax Rate
< \$100,000	0
\$100,000–\$200,000	5% on amount > \$100,000

\$200,000–\$300,000	\$5,000 + 15% on amount > \$200,000
\$300,000–\$400,000	\$20,000 + 30% on amount > \$300,000
> \$400,000	\$50,000 + 50% on amount > \$400,000

Table 2.2 illustrates how this convex tax schedule makes hedging valuable by reducing the average taxes the firm must pay. If the firm is unhedged, then in each year there is a 50% chance that its taxable income will either be \$150,000 or \$450,000. Given the foregoing convex tax schedule, the firm will pay \$38,750 per year in taxes on average. But if the firm can hedge so that its taxable income every year is the average of its unhedged yearly taxable income (i.e., \$300,000 with certainty), it will pay only \$20,000 per year in taxes every year. Thus, when the tax schedule is convex, hedging can reduce the firm's average taxes.

The next question to ask is obvious: Is the U.S. tax schedule (or any other country's) convex? The answer is not so obvious. Merely examining marginal or average tax rates that apply to different levels of taxable income is not sufficient to permit a conclusion because items such as tax loss carryforwards, investment tax credits, and the alternate minimum tax (AMT) provision can induce convexity in tax schedules. The degree of convexity in tax schedules in the United States is subject to debate, and many models suggest that greater convexity exists than is evident by simply viewing average tax rates. Convexity may exist for some firms and not for others.

2.3.8 Risk-Averse Managers Will Prefer to Hedge

Suppose we argue that stockholders should be indifferent to the firm's **hedging** decision. We might still conclude the firms will hedge because their managers are **risk-averse**. They have a substantial

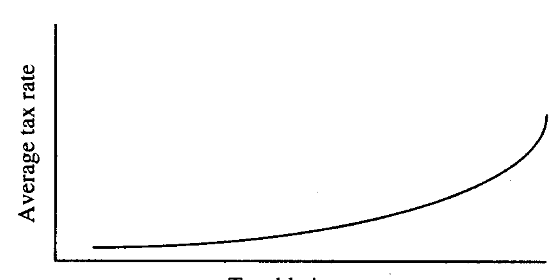

Figure 2.5 A convex tax schedule.

TABLE 2.2 How Hedging Is Valuable When the Tax Schedule Is Convex

Probability	Taxable Income	Taxes
Unhedged		
0.5	\$150,000	\$2,500
0.5	\$450,000	\$75,000
	Expected taxes:	\$38,750
Hedged		
1	\$300,000	\$20,000

amount of human capital and wealth invested in their firms. A bad draw from the probability distribution of the firm's operating income might result in a reduction of their pay or in their being fired. In fact, the bad draw might even be due to factors beyond their control. All their prior decisions might have been "correct" given all available information that was available at the time. But totally unpredictable events might subsequently occur that could lead stockholders to replace the firm's management. Some of management's actions might result in lower profits in the short term, but higher profits in the future. It would be unfortunate if managers failed to make these good decisions because of the possible impact of short-term results on their employment.

Thus, all else equal, we can conclude that managers will often prefer to hedge their firm's activities because of their own risk aversion. If a firm's owners are indifferent about the managers' risk management activities, the firm should hedge. Even if the firm's owners don't want the firm to hedge, they should think of any risk management activities that risk-averse managers undertake as a different form of compensation because the owners are bearing excessive amounts of firm-specific risk.

2.3.9 Other Reasons for Using Derivatives

Sections 2.3.1–2.3.8 explained why firms should manage their risk exposures. Now we add some other reasons, not based on risk management, that will enter a firm's decision to use derivatives.

Derivatives can be used to lower interest costs. Later, we will demonstrate that by issuing bonds denominated in a foreign currency and also entering into a swap in which it pays dollars and receives the foreign currency, a firm may have a lower interest expense than would have obtained if it had issued dollar-denominated debt only. A firm that desires fixed-rate debt may find it cheaper to issue floating-rate debt and engage in a swap so that it creates synthetic fixed-rate debt.

Derivatives are useful because they give the firm flexibility. By using derivatives, firms can alter their assets and liabilities, and their revenues and expenses, more quickly and with less cost than is possible by dealing with actual assets and liabilities.

Arbitrage profits can be earned by some firms when they perceive mispricing in different derivative markets.

Finally, some firms and institutions wish to accept some price risks. Another word for this is *speculation*. Derivative contracts are low cost, liquid conduits for price speculation. Unfortunately, some firms have not had adequate controls on some of their employees, who proceeded to incur staggering losses by speculating with derivatives. But many other firms have successfully speculated, and their activities are useful to society in that they are willing and able to bear these risks. Remember that if one firm is bearing these risks, it is quite possible that some other party has shed itself of them.

2.4 What Should Be Done After Risk Exposures Have Been Identified?

Other decisions remain even after a firm's managers have estimated how and to what extent their company's value is impacted by changing prices.[12]

One important decision that upper management must make is whether the risk management group should be allowed to speculate by selectively hedging the firm's exposures, or whether the goal is to always minimize the firm's risk exposure by maintaining a continuously hedged position.

In the first case, risk management activities will be undertaken only when it is forecast that prices will move in an adverse manner. The firm would then be unhedged when the forecast is that price changes will increase firm value and profits. Most firms selectively hedge their risk exposures (see Dolde, 1993).

Selectively hedging risk exposures is tantamount to speculation. Imagine a selective hedger who is long a cash asset and is always wrong about forecasted price movements. Whenever he believes prices will rise, he maintains an unhedged long position in the cash asset. But prices always subsequently fall. Whenever he believes that the value of his spot position is going to decline, he hedges with the sale of futures contracts. Inevitably, however, prices subsequently rise, and the gains on the cash asset are offset by losses in the futures market. This selective hedger will not have much in the way of assets before long!

A selective hedger should always monitor his performance against two benchmarks: a continuously hedged position and an unhedged position. If markets are efficient, then after accounting for transactions costs and the time needed to analyze market conditions, the selective hedger probably will underperform the continuous hedger, unless the selective hedger has exceptional skills at forecasting future prices.

Brown, Crabb, and Haushalter (2001) analyze the risk management policies of 48 firms from three different industries. They conclude that these firms practice selective hedging, but that few are successful, and any gains are quite small.

Management must be sure to supply sufficient resources to create a knowledgeable and well-trained risk management group. Objectives must be clearly defined. Price data must be supplied. Record-keeping systems must be purchased. Internal controls must be put into place. Performance must be monitored. As part of implementing the risk management plan, guidelines must be established for deciding which tools to use. One method of hedging involves the use of the spot market. These acts, which might be called operational hedges, require that the firm create spot, or physical, assets and liabilities to hedge. For example, many Japanese auto manufacturers built plants in the United States to be able to hedge against the risk of a decline in the yen value of the U.S. dollar. Alternatively, the firm will use the derivative contracts described in Chapter 1.

2.5 Accounting for Derivatives: FAS 133

The rules that define the accounting conventions for derivatives positions are complex. Different rules apply, depending on the purpose of the derivatives. The general practice for traders (speculators) and market makers is to mark all derivatives to market at the end of the accounting period. All profits and losses, regardless of whether they are realized or unrealized, are included in a firm's income statement, if the derivatives are not used for hedging.

On June 15, 1998, the Financial Accounting Standards Board (FASB) issued Financial Accounting Standard No. 133, "Accounting for Derivative Instruments and Hedging Activities" (FAS 133). FAS 133 has applied to all firms that use derivatives for hedging since the accounting period beginning after June 15, 2000. Firms were permitted to adopt the new rule as early as July 1, 1998.

FAS 133 establishes three new classifications of hedges, each with its own accounting treatment. Hedge accounting for derivatives applies only if the hedge meets the definition of one of these three categories. Moreover, procedures must be in place to document and determine the effectiveness of the hedge. The three classifications are as follows.

- **Fair value hedges** exist when the derivatives are used to hedge the risk resulting from changes in the value of an asset, liability, or unrecognized firm commitment (a binding agreement to enter into a transaction with an unrelated party). Both the derivative and the item being hedged are marked to market, and the changes in the value of both are recognized in the firm's net income. If the hedge is effective, the change in the gain (loss) in the value of the derivative will equal the loss (gain) in the value of the item being hedged; then, there will be no net impact on the firm's earnings. A firm that synthetically converts a fixed-rate liability into a variable-rate liability through an interest rate swap is creating a fair value hedge. Another fair value hedge is created when a firm that uses interest rate futures contracts to hedge changes in the value of a fixed-rate debt instrument that is available for sale.
- **Cash flow hedges** are used to hedge forecasted transactions. Firms face risks from changes in the amount of future cash flows that will emerge from recognized assets and liabilities (e.g., interest payments on floating-rate debt or interest income on loans) or forecasted transactions. The key is that the prospective cash flow is uncertain. If the specified criteria for a cash flow hedge are met, an institution can use a derivative to try to "lock in" the amount of a future cash inflow or outflow. Using an interest rate swap to synthetically convert the interest payments on a variable-rate loan to a fixed amount is an example of a cash flow hedge; using a forward rate agreement (FRA) to lock in the interest rate on the forecasted issuance of fixed-rate debt is another. The derivative results must be separated into "effective" and "ineffective" parts. The effective portion of the change in fair value of the derivative is initially recognized in a separate component of equity called "other comprehensive income." Then, in the period during which the forecasted cash flow affects earnings, the effective portion is reclassified as income. The ineffective portion of the hedge is reported in earnings (current income).
- **Foreign currency hedges** protect a firm from the risk resulting from changes in foreign currency values. FAS 133 also defines these as "net investment hedges," which apply to the net investment in a foreign operation. If specified criteria are met, a firm can use derivatives in a foreign currency fair value hedge or cash flow hedge. A firm that uses derivatives to "lock in" the U.S. dollar cost of a foreign currency transaction would be considered to be creating a "foreign currency" hedge. Or, a firm might use a forward contract to hedge the amount of a cash outflow on an anticipated foreign currency–denominated transaction, or purchase a put option to hedge changes in the fair value of a foreign currency–denominated debt security (an asset). The accounting for a foreign currency hedge depends on the transaction, but its treatment will be similar to that of either a fair value hedge or a cash flow hedge.

It is important to recognize that firms are required to establish the effectiveness of derivative hedges. Firms must measure and monitor performance at least quarterly. Knowledge of statistics is essential to accomplish this. Also, derivative hedge transactions must be documented when they first occur. FAS 133 is a very complicated document. Consult an expert before you venture into hedge accounting with derivatives. A website devoted to helping treasurers, risk managers, accountants, and others understand and implement FAS 133 is www.fas133.com/.

It is no secret that neither firms nor investors like earnings volatility. Both prefer steadily increasing earnings that are predictable. The rules that dictate how firms must account for their transactions with derivatives do affect their income statements. These rules affect firms' decisions whether to hedge. If they do hedge, they affect the decision about how to manage risk. It is unfortunate that what should be a rational decision based on economic reality often becomes a poor decision made for cosmetic accounting reasons. In other words, accounting rules affect risk management decisions.

2.6 SUMMARY

This chapter provides an overview of the nature of risk. In this chapter, we focus on the questions of what risk is, how risk is measured, how risk exposure can be determined, and the factors that should induce firms to manage risk.

Risk exists because the future is uncertain. In finance, uncertain future outcomes such as prices and rates of return are called random variables; random variables are characterized by lists of possible outcomes and their associated probabilities. The variance, or standard deviation, of the distribution from which prices or rates of returns are drawn measures the risk of that variable. The greater the variance, the greater the risk or uncertainty about what the outcome will be. In this chapter, we reviewed the computation of the variance of a random variable and explained how historical data can be used to estimate risk.

It is important for a firm to ascertain its risk exposures to different prices. One approach is to estimate the historical relationship between the firm's cash flows, or the rate of return on its common stock, to changes in key commodity prices, interest rates, and prices of currencies. The other method requires the use of Monte Carlo simulation to gain knowledge about how the firm's cash flows will be related to future changes in financial prices.

The key reasons that firms should manage risk are as follows: to reduce the level of expected financial distress costs, to maintain an optimal real investment strategy, to exploit superior information about prices (commodity prices in particular, since no firm should be arrogant enough to believe that its has superior information about future interest rates or foreign exchange rates), to hedge the concentrated investment of the firm's owners in the firm's common stock, to reduce expected taxes, and because management is risk-averse.

FAS 133 has applied to all firms that use derivatives for hedging since June 15, 2000. FAS 133 defines how firms must account for their derivatives transactions that are undertaken to manage risk. These rules result in an impact on earnings. Firms that are concerned about their earnings will let accounting rules affect their risk management decisions.

Further Reading

Some key articles that identify market imperfections that provide risk management incentives are as follows. See also Froot, Scharfstein, and Stein (1993, 1994).

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Several articles have examined the actual nature of firms' risk management activities. Some recent examples are as follows. See also Bodnar, Hayt, and Marston (1998) and Dolde (1993).

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Tufano, Peter. 1996. "Who Manages Risk? An Empirical Examination of Risk Management Practices in the Gold Mining Industry." *Journal of Finance*, Vol. 51, No. 4, September, pp. 1097–1138.

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Bauman, Joseph, Steve Saratore, and William Liddle. 1994. "A Practical Framework for Corporate Exposure Management." *Journal of Applied Corporate Finance*, Vol. 7, No. 3, Fall, pp. 66–72.

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Notes

[1]There are many websites devoted to financial engineering and enterprise risk management. Any search engine will turn up hundreds. Some good starting points are www.garp.com, the home page of the Global Association of Risk Professionals (GARP), www.margrabe.com/, the online magazine about derivatives, the search engine at www.financewise.com/, and www.erisks.com/, the homepage for Erisk.

[2]Indeed, a change in the output price will likely affect the quantity that a firm sells.

[3]Some companies now sell weather derivatives, which offer payoffs that are contingent on the weather (rainfall amount, temperatre, etc.).

[4]Interest rates are prices, too. An interest rate is the price of current consumption. By consuming today, an individual is foregoing future consumption, since anything spent today cannot be invested to earn interest, thereby providing more (at least in nominal terms) consumption at a later date.

[5]A tilde (~) above a symbol identifies it as a random variable.

[6]Diversification is another method of reducing risk. If the risks possessed by different assets are not correlated, then combining the assets will create a portfolio effect in which variance and standard deviation are reduced.

[7]Note that because it is symmetric, the normal distribution is not skewed.

[8]Frequently, we use another formula for variance, which is known as the maximum likelihood estimator:

$$\frac{1}{N}\sum_{t=1}^{N}(r_t - \bar{r})^2$$

[9]The Postscript to the Banc One Corporation case by Esty, Tufano, and Headley (1994) suggests that corporate managements may have to educate investors about their firms' risk exposures before stock prices begin to reflect those risks.

[10]A perfect capital market is one in which there are no transaction costs (commissions or bid–ask spreads), there are no taxes, there are no costs of financial distress or bankruptcy, individuals cannot affect prices by their trades, and all market participants, including investors and firms, have equal access to information. It is also important to assume that the investment decision has already been determined, and will not be affected by the firm's decision to hedge or not to hedge.

[11]We will not dwell upon the issue of what determines or defines that discount rate, but it could be argued that the discount rate that should be applied to the hedged firm's expected cash flow stream is equal to or lower than the one applied to the unhedged firm's expected cash flows.

[12]See Bauman, Saratore, and Liddle (1994) for a description of a four-step procedure that can establish a corporate risk management framework, including identifying its exposures, formulating management, planning, and policy decisions, implementing policy, and establishing the proper controls and evaluation methods.

PROBLEMS

2.1 What is financial price risk?

2.2 Discuss the factors that induce firms to undertake risk management activities.

2.3 How might a law that changes the tax code to a "flat tax" affect firms' hedging decisions?

2.4 A firm currently sells 1000 gold rings each year. Is it exposed to the risk that gold prices will rise or fall? Compare the nature of its hedging decision (a) if it assumes that demand for its rings is unaffected by changes in the price of gold, and (b) if it believes that demand for its rings is elastic.

2.5 Estimate the expected value, the variance, and the standard deviation for the following random variables.

Price	Probability
$40	0.22
$50	0.58
$60	0.12
$70	0.08

2.6 An energy production firm is trying to decide whether to hedge. It feels that its greatest source of risk exposure lies in whether significant new crude oil discoveries will or will not be made next year. It believes that if none are made, energy prices will rise at a rate of 40%/year for the following two years, and annual earnings before interest, depreciation, and taxes (EBIDT) will be $70 million. But if new discoveries are made, prices will be flat, and EBIDT will be $8 million. The probability that new discoveries will be made is 30%. Interest expense will be $5 million, and depreciation expense will be $12 million. The tax rate is 40%. Compute the firm's expected net income and expected cash flow. Compute the variance of its net income.

Now suppose that if the firm hedges its exposure to changes in crude oil prices, it can reduce the uncertainty about its future EBIDT. Hedging would indicate the belief that if there is no new oil to be discovered, EBIDT will be $55 million; but if there are new discoveries, EBIDT will be $43 million. Compute the firm's expected net income and expected cash flow. Compute the variance of its net income.

2.7 Variance and standard deviation are often criticized as risk measures because they incorporate both downside risk and upside "risk." Discuss this criticism.

2.8 Discuss three ways by which a firm can identify its exposure to different price risks. Identify one good aspect and one drawback for each approach.

2.9 Firms should manage their financial risk exposure for many reasons. Which one of the following is ***not*** a reason:

a. To increase the firm's cost of capital
b. To lower taxes
c. To reduce the costs of financial distress
d. To increase debt capacity
e. To increase the likelihood that the firm will adopt good investment projects in the future

2.10 Central tendency, dispersion, and symmetry can be used to describe a probability distribution. Which of the following alternatives offer correct terms to describe each of these attributes?

Central Tendency	Dispersion	Symmetry
a. mean	median	skewness
b. variance	standard deviation	correlation
c. mean	standard deviation	skewness
d. median	variance	correlation
e. mean	skewness	correlation

2.11 Which one of the following tools can be used to measure or identify a firm's risk exposure to changing interest rates?

a. Estimating a regression model with the change in interest rates as the dependent variable and the rate of return on the market as the independent variable, as in

$$\Delta(\text{int rate}_t) = a + b\,\Delta\text{RMKT}_t + \varepsilon_t$$

b. Estimating the firm's beta.

c. Using Monte Carlo simulation.

d. Determining the firm's debt capacity.

e. Determining whether the firm practices selective hedging or continuous hedging.

2.12 There are many reasons for a firm to manage its financial risk exposure. Which of the following is ***not*** a reason:

a. Risk management can increase the amount of funds that the firm can borrow.

b. Risk management can lead a firm to accept good investment projects.

c. Risk management should be performed by firms when they (the firms) have better information than individuals.

d. Risk management is beneficial when the firm's owners have a substantial amount of their own wealth invested in the firm.

e. Risk management can increase taxes when the tax schedule is convex.

A Project

a. Select a publicly traded firm (preferably a large one), obtain its most recent annual report, and determine whether the firm does business in a foreign country (either as an importer or as an exporter or has foreign-based manufacturing facilities). Also determine whether it has exposure to any commodity prices. Try to predict the direction of the risk exposure (e.g., Eastman Kodak might be exposed to the risk that silver prices will rise because silver is a key input to its products, to the risk that the Japanese yen will decline in value because this would help its Japanese-based competitors export to the United States, and to the risk of rising interest rates because it has a great deal of floating-rate debt outstanding).

b. Obtain 36 months of the stock's historical rates of return, changes in short-term and/or long-term interest rates, percentage changes in the prices of the foreign currency to which it seems to be most exposed, and percentage changes in the commodity price to which it seems to be most exposed.

c. Define r_i as the rate of return on the stock, int as the interest rate, *fx* as the currency to which it is most exposed to risk, comm as the commodity price to which it is most exposed to risk, and r_m as the rate of return on a broad market index such as the S&P 500 or the Wilshire 5000. Estimate the following regression model using the 36 months of historical data:

$$r_i = b_0 + b_1\Delta\,\text{int} + b_2\%\Delta fx + b_3\%\Delta\text{comm} + b_4 r_m + \varepsilon$$

Interpret the meaning of your estimated regression coefficients.

There is no guarantee that your estimated slope coefficients will confirm your predictions, for several reasons. The firm may be managing its risk exposure. Its risk exposures two and three years ago may have differed from last year's level. The magnitude of its risk exposures may be small. Its risk exposures may be systematic (part of market risk) and picked up in the estimated b_4 coefficient. The impact of changing prices may be reflected in the firm's stock price with a delay.

PART 2

FORWARD CONTRACTS AND FUTURES CONTRACTS

CHAPTER 3

Introduction to Forward Contracts

The forward contract is the most basic derivative contract. A forward contract is an agreement to buy or sell something in the future. The agreement is made today to exchange cash for a good or service at a later date. This differs from a spot transaction, which is the usual way of buying or selling something. In a spot transaction, one party pays for a good or service, and immediately receives that good or service.

This chapter first describes the general concepts of forward contracts. Then, greater details are presented concerning two types of forward contract that are prevalent in modern business operations and are used to manage financial price risk: the forward rate agreement (FRA), which is an arrangement to borrow or lend money at a future date at an agreed upon interest rate, and the forward foreign exchange contract, in which a party agrees to buy or sell an amount of a foreign currency at a future date.

3.1 General Concepts

When a forward contract is created, there must always be two parties: the buyer and the seller. The buyer of a forward contract agrees to buy something in the future. The buyer is also said to have a long position, or be long the forward contract. The seller of a forward contract has the obligation to sell something in the future, and is said to have a short position.

The terms of the contract are agreed upon today, and delivery and payment take place in the future, at what is called either the **delivery date**, the **settlement date**, or the **maturity date** of the contract. The buyer has agreed to take delivery and the seller has agreed to make delivery.

Money rarely changes hands when a forward contract is originated (unless one or both of the parties demands "good faith" money to serve as collateral that backs up the obligations stated in the contract). Payment from the buyer of the forward contract to the seller is generally made only upon the delivery of the good.

Most business transactions are actually forward transactions. A firm might order 10,000 widgets from another firm. The price is agreed upon today. No cash flows occur today. The widgets will be delivered one month hence. Payment is not made until after the widgets have been received. For all practical purposes, this is a forward contract.

The failure of a party to do what has been agreed to, as stated in the contract, is known as default. On the day that a forward contract is originated, both parties face potential **default risk**, the most extreme form of **credit risk**, which describes the future uncertainty concerning the other party's ability and/or willingness to fulfill the terms of the contract.[1] For most everyday forward transactions we enter into, there are no penalties for defaulting. If you fail to show up at a restaurant at which you have made a reservation, it is extremely rare that you will be sued or forced to pay for the meal that you promised to, but did not, consume.[2] Penalties for failing to fulfill the terms of forward contracts vary, however, and in business default is a serious matter that will likely lead to legal action.

We are interested in forward contracts to buy or sell a specific good on a specific future date, at a specific price. This price is called the **forward price**. Thus, a firm may agree today to buy 100,000 barrels (bbl) of oil 6 months hence, at \$31/bbl. The forward price is \$31/bbl. The forward price will likely differ from the **spot price**, which is today's price for delivery of oil today. There may be many forward prices, one for each possible delivery date. For example, the forward price for delivery 9 months hence may be \$32/bbl. Forward prices for most physical commodities will also depend on the delivery location; the forward price for delivering crude oil to a site in Maine will almost certainly be higher than the price for delivery that takes place in Texas.

A fair forward price will result in a forward contract that has no value when it is originated. If the buyer and the seller agree that the forward price is fair, then neither party will have to make a cash payment to the other on the initial agreement date. Even though the spot price of oil is \$30/bbl, the parties can agree that \$31/bbl is a fair price to pay for oil that will be delivered 6 months hence. The forward buyer will pay the forward seller for the oil on the delivery date. It is important to understand that the equilibrium forward price is a fair price in the sense that the demand for forward contracts equals the supply of forward contracts at that forward price, and the *value* of a forward contract at that fair forward price is zero.

Subsequently, the forward contract will likely become valuable for only one of the two parties. For the party that has a long position, the contract will have a positive value, or become an asset, when the forward price (for delivery on the settlement date of the original contract) rises.

For example, suppose that today is November 26, 2000, and you agree to buy oil 6 months hence, on May 26, 2001, at a price of \$30/bbl. The next day (November 27, 2000), \$30.20/bbl is the fair price for delivery of oil on May 26, 2001 (note that this is no longer a 6-month forward contract; it is a forward contract for delivery 6 months *less one day* in the future). Your long position is now valuable. You have a contract that entitles you to buy oil at \$30/bbl. But new agreements, being originated on November 27, have a forward price of \$30.20. Your contract to buy oil at only \$30/bbl is "a bargain." It is a valuable asset for you.

It then follows that the forward contract to sell oil at \$30/bbl has become a liability for the counterparty to the forward contract. The seller of the forward contract is obligated to deliver oil on May 27, 2001, at only \$30/bbl, while new contracts are being created to sell oil at a higher price. *Forward contracts, like all derivatives, are zero-sum games.* Whatever one party gains, the other party must lose. Thus the party that agreed, on November 26, to sell you oil at a price of \$30/bbl has, on November 27, a contract with negative value; this party who is short the contract has a liability because the forward price rose.

Table 3.1 summarizes how the parties that are long and short a forward contract profit or suffer losses when forward prices change from the original forward price that was agreed upon in the original contract.

The profits and losses associated with forward contracts are typically realized at delivery. Before delivery, as forward prices for delivery on the settlement date of the original contract fluctuate, each party could experience unrealized gains and losses. For either party, the forward contract may change from being an asset on some dates to being a liability. But the actual profit or loss is realized only on the delivery date.

TABLE 3.1

Position	Profits	Loses
Long a forward contract	Forward prices rise	Forward prices fall
Short a forward contract	Forward prices fall	Forward prices rise

Let us define the following:

Origination date of the forward contract	time 0
Delivery date of the forward contract	time T
Forward price on the origination date of the forward contract	$F(0,T)$
The spot price on the delivery date	$S(T)$

The actual profit or loss for the party that is long the forward contract is then $S(T)-F(0,T)$ per unit of the good under contract. If the spot price on the delivery date is greater than the original forward price, then this is positive, and the long position realizes a profit. If the spot price at delivery is less than the original forward price, the long suffers a loss.

The actual profit or loss for the party that is short the forward contract is the same amount, but the opposite sign: $F(0,T)-S(T)$. Whatever profit the long realizes, the short must lose. If the seller of the forward contact makes a profit because the spot price at delivery is less than the forward price specified on the origination date $[S(T)<F(0,T)]$, the buyer of the forward contract must realize a loss.

In our example of the forward oil contract, $F(0,T)=\$30$/bbl for 100,000 bbl of oil. Suppose that at delivery, the spot price of oil is $S(T)=\$28.61$/bbl. Then, the long realizes a loss of \$1.39/bbl × 100,000 bbl = \$139,000. The long is contractually obligated to buy 100,000 bbl of oil at the originally agreed-upon forward price of \$30/bbl. This is higher than the prevailing spot market price of oil on the delivery date. Effectively, the long is forced to overpay for the oil on the delivery date.

The per-barrel profit for the short is $F(0,T)-S(T)=30-28.61=\$1.39$/bbl of oil. The profit arises because the seller earlier contracted to sell the oil at a price that turned out to be higher than the spot market price. Multiply this price difference by the amount of oil specified in the contract, and we obtain the result that the short realizes a profit of \$139,000.

Many forward contracts are **cash settled**. This means that no delivery takes place on the settlement date. Instead, the party with the profitable position receives a cash payment from the party with the unprofitable position. The cash payment equals $|F(0,T)-S(T)|$, where the vertical lines are used to denote "absolute value."

Another device for describing how profits and losses occur for long and short forward positions is the profit diagram. The profit diagram for a long forward position is presented in Figure 3.1. The profit diagram for a party that is short a forward contract is shown in Figure 3.2. The profit line for the long position is a positively sloped 45° line. The long's profit increases by a dollar for every dollar that the delivery day spot price exceeds the original forward price, times the number of units of the good that the contract covers. The short's profit is shown as a negatively sloped 45° line. If $S(T)$ is a dollar less than $F(0,T)$, the short's profit is a dollar times the number of units of the good that the contract covers.

Earlier, it was stated that when a forward contract is originated, it has no value. At subsequent times, it will almost surely have positive value for one party and negative value for the other party.

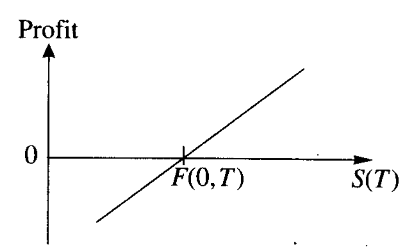

Figure 3.1 Long forward profit diagram: the long profits if the spot price at delivery $S(T)$ exceeds the original forward price, $F(0, T)$.

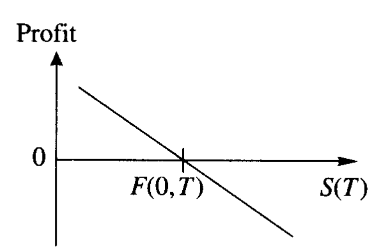

Figure 3.2 Short forward profit diagram: the short profits if the price at delivery $S(T)$ is below the original forward price, $F(0, T)$.

When a forward contract becomes an asset (has positive value) for one party, that party will become concerned about the other party's ability and willingness to fulfill the contract's terms. Many different terms are used to represent the condition of concern: **default risk, performance risk**, or **credit risk**. At a point in time, only one party, the one for which the forward contract is an asset, will worry about *current* default risk. If the losing party reneges on its obligation, the winning party will not be able to realize the contract's value.

Thus, if $F(0,T)=30$, and at some later date at time t prior to delivery ($0 < t < T$), $F(t,T)=31$, the long position has an unrealized profit and will be concerned about whether the party who is short the forward contract will go bankrupt, or just refuse to abide by the terms of the contract at delivery.[3] This fear about current performance is only one sided at time t. The short does not fear that the long will go bankrupt at time t and default on its obligation because the value of the short's position at that time is negative. Given the information about prices that it has as of time t, the short would not mind wiping a liability from its economic balance sheet.

Thus, only one party at one point in time fears that its counterparty will actually default at delivery. Of course, since forward prices change over time, the party exposed to default risk can also change over time. Finally, it should be understood that forward contracts can frequently build up a great deal of asset value for one of the parties (and because they are a zero-sum game, they will therefore become a sizable liability for the counterparty).[4]

Suppose that a party has entered into a forward contract to buy oil and before delivery (time t), it decides that it no longer wants this long position. The ability to quickly buy and sell commodities, securities, and derivative contracts at fair prices is called **liquidity**. Liquidity is a highly desired and valued aspect of any contract and good. The party that is long the forward contract has two ways of ridding itself of the contract. First, it can approach the other party and negotiate its early termination. In this case, the party that has a losing position at time t will have to pay to get out of its obligation. If $F(t,T) > F(0,T)$, then the short will have to pay the long to cancel the forward contract. If $F(t,T) < F(0,T)$, then the long is losing money because the forward price has declined, and he will have to make a payment to the short.

Alternatively, the party who is long the contract, and no longer wants to abide by its terms, can approach a third party and agree to *sell* the good forward, at the fair forward price on day t, $F(t,T)$. Then, this forward selling party has both the obligation to buy the good at time T [at a price of $F(0,T)$], and the obligation to sell the good at time T [at $F(t,T)$]. This new forward contract effectively offsets the original contract. At time T, the long will realize the profit if $F(0,T) < F(t,T)$, because he will buy the good for a lower price than the one he will receive. The long will have to make a payment at delivery if $F(0,T) > F(t,T)$ because the original agreed-upon purchase price is above the price at which he (later) agreed to sell the good.[5]

Businesses around the world transact in forward markets. Forward agreements to buy and sell debt securities issued by most governments of developed nations, such as U.S. Treasury securities, are commonplace. There are well-developed forward markets in many energy products such as crude oil, and precious metals such as gold. Farmers often sell their products before harvesting, to the users of these agricultural products. Forward contracts do not trade on organized exchanges. Instead, firms usually trade with financial institutions that make markets in forward contracts. This market is often called the **OTC market**, or the **over-the-counter market**.

Table 3.2 presents the 10 U.S. bank holding companies with the largest positions in OTC forward contracts, as of December 31, 2000. These 10 banks alone had over 98% of the total notional amount of forward contracts held by all the commercial banks in the United States. Chase Manhattan Corp. has the largest position in forward contracts: almost $3 trillion.

Sections 3.2 and 3.3 describe in great detail two important forward contracts, those for the purchase and sale of interest rates and foreign currencies.

TABLE 3.2 Notional Amount of OTC Forward Contracts Held by Leading U.S. Bank Holding Companies, December 31, 2000 ($ Millions)

Chase Manhattan Bank	$2,813,869
Citibank	$1,848,417
Bank of America	$1,238,559
Morgan Guaranty Trust Co. of New York	$1,084,561
State Street Bank & Trust Co.	$133,895
First Union National Bank	$131,436
Bank One National Association	$104,627
HSBC Bank USA	$90,268
Fleet National Bank	$42,668
Bank of New York	$40,347
Total notional principal of forward contracts for the top 25 holding companies with OTC forward positions	$7,634,246

Source: Website for the Office of the Comptroller of the Currency (www.occ.treas.gov/).

3.2 Forward Rate Agreements

A forward rate agreement (FRA) is a forward contract in which the buyer effectively promises today to borrow an amount of money in the future, at an agreed-upon interest rate. This rate is called the forward rate. The seller of a FRA has the obligation to lend money at the forward interest rate. You should think of the FRA as an agreement for the long to buy an interest rate; if the interest rate rises, the long profits. The long has agreed to borrow money at the original forward rate, and if interest rates subsequently rise, this original forward rate will turn out to be low. Borrowers prefer to borrow at low rates, and thus, the buyer of the FRA "wins" if interest rates rise. The seller of the FRA, like the seller of any forward contract, will realize a profit if the price falls. In this case, the price is the interest rate, so the party who is short a FRA will profit if the spot interest rate at settlement is less than the originally agreed upon forward rate; the short receives a cash payment at maturity when interest rates have declined. The payoffs for FRAs are depicted in Figures 3.3 (for the party that is long the FRA), and 3.4 (for the party that is short the FRA)..The symbol for the change in interest rates is Δr.

Because FRAs are cash settled (i.e., money is not actually borrowed or lent at settlement), on the settlement date, the winning party pays the losing party.

Because FRAs have *three* relevant dates, the following special notation is required:

Origination date of the forward contract	time 0
Delivery date of the forward contract (start of the forward period)	time $t1$
The end of the forward period	time $t2$
Forward rate on the origination date of the forward contract	$fr(0, t1, t2)$
The spot rate on the delivery date	$r(t1, t2)$
The length of the loan period	$t2 - t1$

More explanation of this notation is needed. At time 0, the buyer of the FRA agrees to borrow money from time $t1$ until time $t2$, at an interest rate of $fr(0, t1, t2)$. The spot interest rate will not be known until time $t1$. The length of time for the forward loan, $t2 - t1$, is called the **loan period.**[6] It is the length of time during which money will effectively be borrowed or lent.[7]

Figure 3.5 is a time line that illustrates the difference between the three dates that are important for understanding FRAs: the origination date; the settlement date or delivery date,

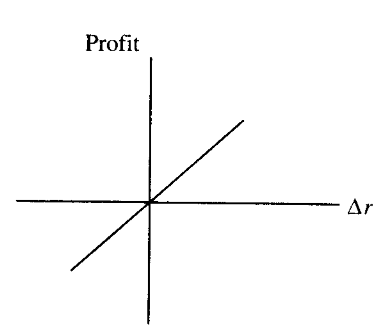

Figure 3.3 The party that is long a FRA profits if interest rates rise (i.e., if Δr is positive).

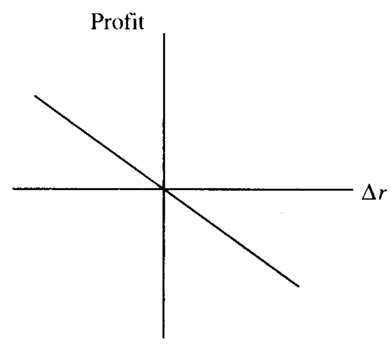

Figure 3.4 The party that has sold a FRA profits if interest rates decline (i.e., Δr is negative).

Figure 3.5 Important dates in a FRA.

which is the start of the forward period; and the end of the forward period. The loan period is the time during which the principal amount is effectively either borrowed or lent.

The interest rate that is covered by the FRA is typically (but definitely not always) a LIBOR[8] rate, and it is specified in the agreement.

While a FRA is originated at time 0, the typical jargon is to refer to it as a $t1 \times t2$ FRA, a $t1$ vs $t2$ FRA, or a $t1$ V. $t2$ FRA, where $t1$ is the starting date of the forward loan and $t2$ is the ending date of the forward period. For example, a 2×5 FRA is an agreement to borrow or lend for a 3-month period, beginning 2 months hence. Verbally, it is stated as "two month against five month," or "2s against 5s." A 1×7 FRA is a 6-month forward rate, beginning 1-month hence and ending 7 months hence; the loan period is 6 months.

Hundreds of banks around the world quote and trade FRAs, and the market is quite liquid. Bid–asked spreads are very narrow: as little as 3 or 4 basis points. Thus, for a 6×12 FRA, a bank might be willing to borrow at a rate of 5.92% for a 6-month period, 6 months hence, and lend at a rate of 5.97%; this is a spread of 5 basis points.

Payment is made on the FRA's settlement day, which is at time $t1$. The payment amount, π, is determined as follows. The spot interest rate at time $t1$ for the loan period specified in the FRA, $r(t1, t2)$, is observed and subtracted from the original forward rate, $fr(0, t1, t2)$. This difference is then "unannualized" to reflect the loan period, and multiplied by the principal amount of the FRA. Then, that amount is discounted, because payment is made in advance (at the start of the loan period), at time $t1$. If we use the following nomenclature:

Principal amount of the FRA	P
Number of days in a year (either 360 or 365, as specified in the FRA)	B

Number of days in the loan period D (=$t2-t1$, in days)

Fraction of year made up by loan period D/B

Then, the payment amount is:

$$\pi = \left| \frac{P[r(t1,\ t2) - fr(0,\ t1,\ t2)](D/B)}{1 + [r(t1,\ t2)(D/B)]} \right| \tag{3.1}$$

The numerator of Equation (3.1) is the dollar profit or loss on a loan that was originally contracted at a forward rate of $fr(0,t1,t2)$ when the actual spot rate turns out to be $r(t1,t2)$. Since both these are annual rates, they must be converted to periodic rates by multiplying them by the fraction of the year for the forward loan, D/B. This periodic rate difference is then multiplied by P, the principal amount of the FRA. The denominator of Equation (3.1) is a discounting factor that brings the numerator to a present value amount. The payment is made at time $t1$, the settlement day of the FRA.

Almost always, the number of days in the loan period, D, is the actual number of days between dates $t1$ and $t2$. Thus, a 2×5 FRA may actually have 89, 90, 91, or 92 days in the loan period. The convention for most U.S. dollar FRAs is to define the number of days in a year as $B=360$. The convention for British pound FRAs is to define $B=365$. But either definition of a year is possible in either country, since FRAs are custom-made contracts.

If $r(t1,t2)$ exceeds $fr(0,t1,t2)$ then the long position (the buyer of the FRA) receives the amount computed by Equation (3.1). The seller of the FRA (the short) is the loser and must pay the computed amount. The two vertical lines means Equation (3.1) defines the absolute value of the payment amount.

A simple example illustrates how the formula that computes the payment amount works. On September 2, a bank sells a 2×5 FRA with a principal amount of \$500 million, at a forward rate of 6%. On the settlement date (2 months hence), spot 3-month LIBOR is 4.8%. A year is defined to be $B=360$ days. What is the payment that will be made on the settlement day?

First, note that there are 91 days in this particular loan period ($5-2=3$ months). The payment is made 2 months after the FRA is originated, and the payment amount is computed to be \$1,498,485:

$$\left| \frac{(\$500,000,000)(0.048 - 0.06)(91/360)}{1 + [(0.048)(91/360)]} \right| = \$1,498,485$$

Because the 3-month spot rate at time $t1$ is less than the original contracted forward rate of 6%, the bank will *receive* (a profit of) \$1,498,485, because it sold the FRA.

The payoff to a FRA is discounted (the denominator discounts the profit) because users of FRAs are interested in hedging the interest expense or income on a loan. The actual interest cash flow on the item being hedged is typically made or received at the *end* of the loan period. For example, a firm may be planning to issue 6-month commercial paper 5 months hence. If interest rates do rise, it will have to pay added interest expense 11 months from today. But the payoff on the 5×11 FRA occurs 5 months from today. To equate any added expense on the commercial paper to the profit from the FRA, the spot 6-month LIBOR prevailing at settlement must be used to discount the FRA's payoff.

TABLE 3.3 A FRA is Analogous to a One-Period Swap

FRA	One-Period Swap
Buyer = long	Pay fixed and receive floating
Seller = short	Receive fixed and pay floating

It is instructive to note now that a FRA is effectively a one-period swap. The buyer of the FRA has agreed to receive a floating interest rate (times the principal amount) and pay a fixed interest rate (times the principal amount). The fixed interest rate in a swap is analogous to the forward rate that is specified in the FRA. If the floating interest rate rises above the contractual fixed rate, the buyer of the FRA receives a net payment on the settlement day. The seller of the FRA has agreed to pay a floating interest rate and receive the fixed interest rate.

Table 3.3 summarizes this analogy.

FRAs exist on all major currencies of the world, including the U.S. dollar, the British pound, euros, and the Japanese yen. Futures contracts also trade on short-term debt instruments denominated in these currencies, and these serve as substitutes for FRAs.

FRAs are used by firms and financial institutions to protect themselves against unexpected changes in interest rates. The parties can use FRAs to lock in a borrowing rate or a lending rate for transactions they will undertake at a future date. By buying a FRA, a party locks in a future borrowing rate. The seller of a FRA locks in the interest rate on a future loan that it will make. Sometimes, FRAs are used to speculate on the future course of interest rates. The next chapter focuses on the usage of FRAs for controlling interest rate risk. Chapter 5 covers the computation of forward interest rates and explains how FRAs are priced.

3.3 Forward Foreign Exchange Contracts

A foreign currency is a good just like any other good. Given the familiar concept of the dollar price of an apple or the dollar price of a computer, you should have no trouble understanding the concept of the dollar price of a British pound ($/£), which is the number of dollars required to buy a British pound. The dollar price of a Japanese yen ($/¥) specifies how many dollars must be exchanged for one Japanese yen.

But currencies can be priced in terms of any other currency. Thus, it is important to understand that traders can buy British pounds (£), and pay for them with euros (€); in this case, the exchange rate is €/£, which is the number of euros required to buy one pound. And if instead, a trader wants to buy euros, he can pay for them with British pounds; the relevant exchange rate is then £/€. The € price of a £ is the inverse of the £ price of a €. Thus, suppose it costs €1.5 to buy 1£. Then, the price of a euro, expressed in terms of pounds, is £0.6667/€; this is the inverse of 1.5: 0.6667 = 1/1.5.

Frequently, you will read in the financial press that "the dollar strengthened yesterday." This means that the cost of buying dollars rose. However, the headline does not make clear which currency or currencies the dollar strengthened against. Perhaps the dollar rose against the yen, from ¥108/$ to ¥109/$. The dollar may have appreciated against most currencies, but depreciated against others. Nonetheless, a strengthening currency is one that rises in price.

A currency that loses value is said to have weakened, or depreciated in value. The fact that the U.S. dollar strengthened from ¥108 to ¥109 means that the yen must have weakened, relative to the dollar. In this case the yen dropped in value from \$0.009259/¥ (1/108 = 0.009259) to \$0.009174/¥ (1/109 = 0.009174). The yen is less valuable on the second day than on the first; the yen became cheaper, in terms of dollars.

All the foregoing discussion dealt with spot exchange rates. However, firms, governments, and financial institutions often want to contract today to buy or sell foreign currencies in the future. One way of doing this is to transact in the forward foreign exchange market, which is also frequently called the forward exchange market.

A forward foreign exchange contract is an agreement to buy or sell a currency at a later date at a specified price; the specified price is the forward exchange rate. The forward price paid must be expressed in terms of a currency different from the one being bought or sold. In other words, it is not sufficient to discuss the forward price of a euro without asking "in terms of what?" Is it the pound price of a euro (£/€), or the U.S. dollar price of a euro (\$/€)?

As with most other forward contracts, there is no payment made when the contract is originated (unless collateral is demanded). On the settlement date, two currencies are exchanged, at the forward exchange rate that was agreed upon at origination. The party that is long the contract profits if the spot exchange rate on the settlement day is greater than the forward exchange rate that was originally set on the origination day. If the party bought Canadian dollars (Can\$) forward, and agreed to pay for them with U.S. dollars (U.S.\$) at a forward exchange rate of U.S.\$0.6845/Can\$, then at delivery, this party must pay out U.S. dollars and receive Canadian dollars. If the forward contract specified that Can\$3 million was to be bought, then the party receives Can\$3 million, and pays U.S.\$2,053,500 at delivery.

Put another way, we can define these terms as follows:

Origination date of the forward contract	time 0
Delivery date of the forward contract	time T
Forward exchange rate on the origination date of the forward contract	$F(0,T)$
The spot exchange rate on the delivery date	$S(T)$
Units of foreign currency covered by the contract	N

Both $F(0,T)$ and $S(T)$ are expressed as the price of one currency in terms of another. For example, $F(0,T)$ might be \$0.01/¥, which is the price of yen in terms of U.S. dollars. In this case, N refers to the number of yen in the contract. Table 3.4 summarizes the profits and losses realized by the buyer and seller of forward foreign exchange contracts.

TABLE 3.4 Profits and Losses for Forward Exchange Contracts

	Outcomes	
Position	$S(T) < F(0,T)$	$S(T) > F(0,T)$
Long forward exchange contract	Loss = $[F(0,T) - S(T)]N$	Profit = $[S(T) - F(0,T)]N$
Short forward exchange contract	Profit = $[F(0,T) - S(T)]N$	Loss = $[S(T) - F(0,T)]N$

The forward exchange market for short-term transactions is very active and is as liquid as the spot exchange market and the FRA market. Spreads are very narrow for nearby delivery dates; they get wider the more distant the delivery date, reflecting less liquid markets. Some forward exchange contracts are cash settled, and others are settled with the actual exchange of currencies.

A forward foreign exchange contract is equivalent to a pair of zero-coupon bonds. One of the zero-coupon bonds is an asset, and the other is a liability, denominated in a different currency. Figure 3.6 illustrates that at time *T*, the settlement date, there is an exchange of currencies. In this example, at time 0, the firm agreed to buy 100,000 euros (€100,000) at a forward exchange rate of ¥116/€. This forward transaction had no cash flow associated with it when the contract was originated. At time *T*, the firm receives €100,000, and pays ¥11.6 million.

Now, compare this forward transaction to a firm that has a zero-coupon German bond on its balance sheet as an asset and a zero-coupon yen-denominated bond on its balance sheet as a liability. The face value of the pure discount German debt instrument is €100,000. The firm also earlier issued a zero-coupon bond denominated in yen, having a face value of ¥11.6 million. Both bonds mature on the same date, at time *T*. The cash flows resulting from the existence of these two zero-coupon bonds, one an asset and one a liability, are exactly equivalent to the cash flows that will be exchanged at time *T* because of the forward foreign exchange contract. Thus, a forward contract to buy €100,000 at a forward price of ¥116/€ is equivalent to having a zero-coupon German bond with a face value of €100,000 as an asset and also having a zero-coupon Japanese bond with a face value of ¥11.6 million as a liability.

Let's consider a bit further the equivalency of (a) a forward foreign exchange contract and (b) two zero-coupon bonds (one an asset in one currency, and the other bond as a liability in the other currency). Note that since the values of the cash flows at time *T* are the same, the cash flows at time 0 must be the same, or else the firm could arbitrage. You know that if the forward rate of ¥116/€ is fair, there is no cash flow for the forward contract at time 0. This means that if the firm did decide to instead issue a zero-coupon bond with a face value of ¥11.6 million, and use the proceeds to buy a zero-coupon bond with a face value of €100,000, then the time 0 prices of the two bonds, when converted to one currency by using the spot exchange rate, must be equal. They must be equal so that there is no residual cash flow at time zero if the firm bought and sold the two zero-coupon bonds, just as there is no cash flow at time zero with the

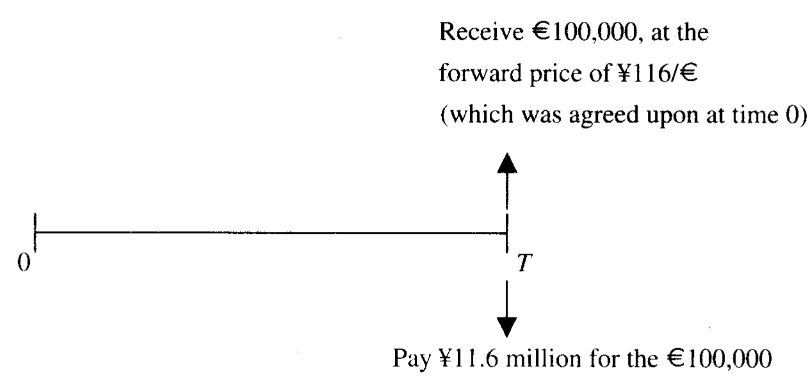

Figure 3.6 Cash flows associated with a forward foreign exchange contract.

forward contract. Put yet another way, the present value of ¥11.6 million must equal the present value of €100,000.

Suppose that the firm could at time 0 buy the zero-coupon German bond maturing at time T for €94,340. Also let the spot exchange rate at time 0 be ¥119.45/€. It must be the case that the firm will receive ¥11,268,913 (€94,340 × ¥119.45/€ = ¥11,268,913) when it issues the zero-coupon bond denominated in yen. If the firm can receive *more* than ¥11,268,913, it should (a) issue the yen-denominated zero coupon bond, (b) take the yen proceeds and convert them to euros at the spot exchange rate, (c) buy the zero-coupon bond denominated in euros, and (d) sell euros forward at the forward price of ¥116/€ (which is equivalent to buying JPY forward at a forward price of €0.00862069/¥, which is the inverse of ¥116/€). This four-step procedure represents arbitrage.

Table 3.5 illustrates the arbitrage process for a firm that can receive ¥11.3 million (which is *more than* ¥11,268,913) when it issues the zero-coupon, yen-denominated bond. Note how the firm has a net cash inflow of €260.21 at time zero, and no cash flow at time one. This illustrates an arbitrage process, which in this case has arisen because the prices violated a relationship that must exist between interest rates, spot exchange rates, and forward exchange rates. You will better understand what the relationship is between spot and forward foreign exchange rates after reading Chapter 5, which covers pricing forward contracts.

Forward foreign exchange contracts are used by firms to lock in the prices they will pay or receive for foreign currencies at future dates. When some firms buy goods abroad, they will have

TABLE 3.5 Illustration of the Arbitrage Process[1]

The situation	Spot exchange rate is ¥119.45/€.	
	Forward exchange rate is ¥116/€.	
	A zero-coupon German bond can be bought for €94,340; face value is €100,000.	
	A zero-coupon, yen-denominated bond can be issued, netting ¥11.3 million; face value is ¥11.6 million.	
Time	**Transaction**	**Cash Flow**
0	a. Issue a zero-coupon, yen-denominated bond (which then becomes a liability for the firm).	+¥11,300,000
	b. Convert the yen proceeds to euros at the spot exchange rate of €0.0083717/¥ (which is the inverse of ¥119.45).	-¥11,300,000
		+€94,600.21
	c. Buy the zero-coupon German bond (which becomes an asset).	-€94,340.00
	d. Sell €100,000 forward at the forward price of ¥116/€.	0
	Net cash flow	+€260.21
1	a. The German bond matures.	+€100,000
	b. Fulfill the terms of the forward contract.	-€100,000
		+¥11,600,000
	c. Pay off the owners of the zero-coupon, yen-denominated bond.	-¥11,600,000
	Net cash flow	0

[1]A forward contract is equivalent to a portfolio of zero-coupon bonds, one an asset and one a liability. If the two are ***not*** equivalent, an arbitrage profit is possible.

to pay for them with the currency the foreign exporter wants, which is typically the exporter's home currency. Other firms that sell goods abroad will often be paid with the foreign currency that was generated when the goods were sold. In general, firms that are involved in foreign trade know that they will have future needs to buy or sell foreign currencies, and forward exchange contracts allow them to manage the risk that the prices of these currencies will change.

Figure 3.7 shows how the *Wall Street Journal* presents spot exchange rates, and also forward exchange rates for some currencies. Note that the U.S. dollar price of each currency is given (in the "U.S. \$ equiv." columns), and the foreign currency price of the dollar is also given (in the "currency per U.S. \$" columns). Thus, on Friday, it cost \$1.4912 to buy one British pound in the spot market. The inverse of 1.4912 is 1/1.4912=0.6706, which is the price of the U.S. dollar denominated in British pounds (£0.6706/\$).

Forward exchange rates are also presented in Figure 3.7 for the British pound, the Canadian dollar, the French franc, the German mark, the Japanese yen, and the Swiss franc. For example, on Friday, a forward contract to trade British pounds 6 months later (for forward delivery on November 26, 2000) had a forward price of \$1.4968/£. (This is above the spot price of \$1.4912/£). All the currencies were selling at a forward premium to the dollar; that is, their forward prices were higher than their spot prices.

THE WALL STREET JOURNAL TUESDAY, MAY 30, 2000 C15

CURRENCY TRADING

Friday, May 26, 2000

EXCHANGE RATES

The New York foreign exchange mid-range rates below apply to trading among banks in amounts of \$1 million and more, as quoted at 4 p.m. Eastern time by Reuters and other sources. Retail transactions provide fewer units of foreign currency per dollar. Rates for the 11 Euro currency countries are derived from the latest dollar-euro rate using the exchange ratios set 1/1/99.

Country	U.S. \$ equiv. Fri	U.S. \$ equiv. Thu	Currency per U.S. \$ Fri	Currency per U.S. \$ Thu
Argentina (Peso)	1.0007	1.0002	.9993	.9998
Australia (Dollar)	.5741	.5690	1.7419	1.7575
Austria (Schilling)	.06770	.06632	14.771	15.078
Bahrain (Dinar)	2.6525	2.6525	.3770	.3770
Belgium (Franc)	.0231	.0226	43.3041	44.2033
Brazil (Real)	.5444	.5414	1.8370	1.8470
Britain (Pound)	1.4912	1.4720	.6706	.6793
1-month forward	1.4919	1.4727	.6703	.6790
3-months forward	1.4937	1.4746	.6695	.6782
6-months forward	1.4968	1.4781	.6681	.6765
Canada (Dollar)	.6650	.6647	1.5037	1.5045
1-month forward	.6655	.6651	1.5027	1.5035
3-months forward	.6664	.6661	1.5005	1.5013
6-months forward	.6680	.6676	1.4971	1.4979
Chile (Peso)	.001909	.001903	523.95	525.50
China (Renminbi)	.1208	.1208	8.2771	8.2773
Colombia (Peso)	.0004748	.0004729	2106.00	2114.50
Czech. Rep. (Koruna)				
Commercial rate	.02559	.02513	39.084	39.791
Denmark (Krone)	.1249	.1224	8.0060	8.1722
Ecuador (Sucre)				
Floating rate	.00004000	.00004000	25000.00	25000.00
Finland (Markka)	.1567	.1535	6.3826	6.5152
France (Franc)	.1420	.1391	7.0416	7.1878
1-month forward	.1423	.1394	7.0269	7.1724
3-months forward	.1429	.1400	6.9980	7.1439
6-months forward	.1438	.1409	6.9546	7.0994
Germany (Mark)	.4763	.4666	2.0995	2.1431
1-month forward	.4773	.4676	2.0952	2.1386
3-months forward	.4793	.4695	2.0865	2.1300
6-months forward	.4823	.4724	2.0736	2.1168
Greece (Drachma)	.002769	.002708	361.16	369.33
Hong Kong (Dollar)	.1283	.1283	7.7921	7.7920
Hungary (Forint)	.003584	.003512	278.99	284.76
India (Rupee)	.02255	.02268	44.350	44.100
Indonesia (Rupiah)	.0001174	.0001183	8520.00	8450.00
Ireland (Punt)	1.1829	1.1587	.8454	.8630
Israel (Shekel)	.2404	.2402	4.1593	4.1631
Italy (Lira)	.0004811	.0004713	2078.55	2121.71

Country	U.S. \$ equiv. Fri	U.S. \$ equiv. Thu	Currency per U.S. \$ Fri	Currency per U.S. \$ Thu
Japan (Yen)	.009336	.009313	107.11	107.38
1-month forward	.009389	.009365	106.51	106.78
3-months forward	.009497	.009473	105.30	105.56
6-months forward	.009667	.009643	103.45	[illegible]
Jordan (Dinar)	1.4075	1.4075	.7105	.7105
Kuwait (Dinar)	3.2510	3.2468	.3076	.3080
Lebanon (Pound)	.0006634	.0006616	1507.50	1511.50
Malaysia (Ringgit)	.2632	.2632	3.8000	3.8000
Malta (Lira)	2.2915	2.2604	.4364	.4424
Mexico (Peso)				
Floating rate	.1048	.1051	9.5400	9.5170
Netherland (Guilder)	.4227	.4141	2.3656	2.4148
New Zealand (Dollar)	.4597	.4535	2.1753	[illegible]
Norway (Krone)	.1118	.1106	8.9414	9.0388
Pakistan (Rupee)	.01927	.01927	51.900	51.900
Peru (new Sol)	.2836	.2845	3.5255	3.5145
Philippines (Peso)	.02328	.02320	42.950	43.100
Poland (Zloty) (d)	.2220	.2206	4.5053	4.5330
Portugal (Escudo)	.004647	.004552	215.21	219.68
Russia (Ruble) (a)	.03535	.03530	28.285	28.325
Saudi Arabia (Riyal)	.2666	.2666	3.7506	3.7506
Singapore (Dollar)	.5767	.5774	1.7340	1.7319
Slovak Rep. (Koruna)	.02169	.02126	46.106	47.086
South Africa (Rand)	.1405	.1396	7.1178	7.1628
South Korea (Won)	.0008795	.0008851	1137.00	1129.85
Spain (Peseta)	.005599	.005485	178.61	[illegible]
Sweden (Krona)	.1109	.1096	9.0206	[illegible]
Switzerland (Franc)	.5952	.5835	1.6802	1.7137
1-month forward	.5972	.5855	1.6744	1.7079
3-months forward	.6009	.5890	1.6642	1.6979
6-months forward	.6063	.5942	1.6493	1.6829
Taiwan (Dollar)	.03245	.03247	30.815	30.800
Thailand (Baht)	.02552	.02555	39.185	39.145
Turkey (Lira)	.00000162	.00000161	617050.00	619630.00
United Arab (Dirham)	.2723	.2723	3.6729	3.6729
Uruguay (New Peso)				
Financial	.08344	.08351	11.985	11.975
Venezuela (Bolivar)	.001463	.001464	683.50	[illegible]
SDR	1.3122	1.3077	.7621	.7647
Euro	.9316	.9126	1.0734	1.0958

Special Drawing Rights (SDR) are based on exchange rates for the U.S., German, British, French, and Japanese currencies. Source: International Monetary Fund.

a-Russian Central Bank rate. Trading band lowered on 8/17/98. b-Government rate. d-Floating rate; trading band suspended on 4/11/00. Foreign exchange rates are available from Readers' Reference Service (413) 592-3600.

Figure 3.7 Sample presentation of spot currency rates in *The Wall Street Journal*. (Reprinted with permission of *The Wall Street Journal*, page C15. © May 30, 2000.).

3.4 SUMMARY

A forward contract is an agreement to buy or sell something in the future. If the forward price is fair, then the value of a forward contract on its origination date is zero. Typically, no money changes hands when the contract is originated. If the forward price rises, then the contract takes on a positive value for the long position, which means that it becomes an asset. If the forward price declines, then the long's position takes on a negative value, and a positive value for the party that is short the contract.

Forward rate agreements (FRAs) are forward contracts on interest rates. The party that buys a FRA has effectively agreed to borrow a principal amount at a specified interest rate (the forward rate). If interest rates rise above the forward rate, this party will realize a profit. Sellers of FRAs will profit if interest rates decline below the forward rate that was set when the contract was originated. The party that sells a FRA effectively agrees to lend a principal amount at a fixed interest rate. Because FRAs are cash settled, this party receives a payment that is defined by Equation (3.1).

Forward foreign exchange contracts are agreements to buy or sell a foreign currency on a future date. The forward exchange rate is the price at which the transaction will occur. The forward exchange rate can be above or below the spot exchange rate. The buyer of a forward exchange contract profits if the spot price of the currency on the settlement date is above the forward price that was agreed upon at origination.

Notes

[1]A party may default for any of several reasons. Usually, a party defaults because it lacks the ability to meet the terms of a contract. But a financially sound party has often defaulted because of a loophole in the contract, or because the defaulting party feels that he has been wronged in some way. "Credit risk" is the term often used to describe the possibility that a party may fall into financial distress and therefore fail to perform the terms of the contract.

[2]It is common for hotels to charge you for a guaranteed room if you fail to notify them before a certain time that you will not be arriving. Some rental car firms have begun to charge customers when they reserve a car (a forward agreement) and then default, or fail to appear at the scheduled time.

[3]For example, time 0 might be November 26, 2000, time t might be March 12, 2001, and time T might be May 26, 2001.

[4]In contrast, futures are settled daily, so they do not build up as much asset or liability value for the two parties.

[5]This example should allow the reader to compute what the long would have to pay or receive if he used the first method to offset his obligation (i.e., negotiated with the original counterparty to let him out of the obligations of the original contract). If the forward price has risen, the long should be paid the present value of $F(t,T)-F(0,T)$. If the forward price has declined, the long's position has negative value, and at time t he should pay the present value of $F(0,T)-F(t,T)$.

[6]It is also called the **contract period**.

[7]We use the term "effectively" because no money is actually borrowed or lent in a FRA; the contract is cash settled. The cash settlement amount is the profit (loss) that would have been realized, had money really been borrowed or lent.

[8]For a definition of LIBOR, see note 9, Chapter 1.

PROBLEMS

3.1 On March 2, a firm sells a 3×8 FRA. The contracted forward rate is 5.93%. The principal amount is $80 million. On the settlement date, the following spot Eurodollar rates are observed:

Maturity	Rate
3 months	5.84%
4 months	5.94%
5 months	6.04%
6 months	6.10%
7 months	6.14%
8 months	6.19%

What amount must the firm pay or receive?

3.2 A firm wishes to get out of a forward contract it agreed to 2 months ago. There are still 3 months until delivery. What are the two methods the firm can use to offset (get out of) its obligation?

3.3 Goldfinger, Inc., buys a forward contract for delivery of 5000 oz. of gold eight months hence. The forward price is $340/oz. Three months later, contracts calling for the delivery of gold 5 months hence have a forward price of $331/oz, and contracts calling for the delivery of gold eight months hence have a forward price of $420/oz. Three months after the contract's origination, is this contract an asset or a liability for Goldfinger? Why?

3.4 On June 6, BilboBank buys a 5×11 FRA at a forward rate of 6.25%, on a principal amount of $40 million. On July 6, it wants to offset its obligation. On that day, the following rates are observed:

FRA	Quoted Forward Rate (%)
4×10	6.06
4×11	6.10
5×10	6.03
5×11	6.14

a. Which of the four FRAs would the bank want to sell to offset its obligation? Why?

b. Suppose BilboBank approaches the original counterparty (the party that originally sold the 5×11 FRA on June 6), and essentially asks if it can sell the FRA back. Will BilboBank have to make a payment to the original counterparty or will BilboBank receive a payment? That is, is Bilbobank making a profit on the original FRA on July 6?

c. How much will Bilbobank *likely* pay/receive for selling the original FRA (see note 5 in this chapter)? Assume that $r(0,4) = r(0,5) = 6\%$ on July 6.

3.5 Let's say that $F(0,T) = \$31/\text{bbl}$ and $S(T) = \$32.20/\text{bbl}$. The forward contract covers 24,000 barrels of oil. Which party, the one that bought the contract, or the one that sold the contract, made a profit? How much is the profit?

3.6 A market maker quotes 3×5 FRAs, denominated in British pounds (GBP, or £) at 8.1% (bid)–8.18% (asked). You buy a FRA with a principal amount of £60 million. At settlement, the following spot rates are observed:

Maturity	Rate (%)
2 months	9.02
3 months	8.84
4 months	8.78
5 months	8.71

By convention, FRAs denominated in GBP use a 365-day year to compute the settlement amount. When will the payment be made? Will you receive a payment at settlement, or will you have to make a payment? How much will the settlement amount be?

3.7 A firm goes long a forward foreign exchange contract. The contract specifies that the firm agrees to buy £5 million 6 months hence. The forward price is ¥168/£. Six months hence, the spot price is ¥180/£.

- **a.** What is exchanged on the settlement day?
- **b.** At settlement, has the firm realized a profit or a loss? What is the amount of that profit or loss?

3.8 A U.S. firm observes that the spot price of Swiss francs is \$0.75/SFR. The forward price for delivery 1-year hence is \$0.79/SFR. U.S. Treasury strips are riskless pure discount bonds. The price of 1-year strips is 93.9% of face value. What must be the price of a zero-coupon SFR-denominated bond that matures 1-year hence?

3.9 Refer to Figure 3.7.

- **a.** If a firm agreed to sell 10 million French francs 90 days hence in exchange for U.S. dollars, then how many dollars would it receive?
- **b.** If the firm instead wanted to receive Swiss francs 90 days hence, use the information in Figure 3.7 to estimate the number of Swiss francs it would receive in exchange for the 10 million French francs.

3.10 Which of the following statements is ***false***?

- **a.** The forward price and the spot price can be different.
- **b.** Forward contracts are zero-sum games.
- **c.** Many forward contracts are cash settled.
- **d.** Most forward contracts trade on organized exchanges.
- **e.** A forward rate agreement is a type of forward contract.

3.11 Which of the following statements is true?

- **a.** Going long a forward contract will always be profitable when the spot price of the underlying asset rises.
- **b.** Going long a forward contract will be profitable when the spot price at delivery is higher than the original forward price.
- **c.** Going long a forward contract is bearish; such a position will be profitable when prices decline.
- **d.** Default risk is low on forward contracts because they are guaranteed by the clearinghouse.
- **e.** A forward contract on foreign exchange is called a FRA.

3.12 What does FRA mean?

- **a.** Futures rate agreement
- **b.** Futures risk agreement
- **c.** Futures rate asset
- **d.** Forward rate agreement
- **e.** Forward risk agreement
- **f.** Forward rate asset

3.13 Suppose that on September 15, 2000, you **sell** €100,000 forward, for delivery on January 15, 2001. On September 15, 2000, the spot price of a € is \$0.95 (i.e., the exchange rate is \$0.95/€), and the forward price for delivery 4 months later is \$0.92/€. Then, on January 15, 2001, the spot price of a € is \$0.93, and the forward price for delivery 4 months later (i.e., for delivery on May 15, 2001), is \$0.94/€. Did you profit or lose on this forward contract, and how much?

a. Profit of $3000

b. Profit of $2000

c. Profit of $1000

d. Loss of $1000

e. Loss of $2000

f. Loss of $3000

3.14 The buyer of a FRA will profit when

a. Interest rates rise

b. Interest rates decline

c. The price of a foreign currency rises

d. The price of a foreign currency declines

e. Stock prices rise

f. Stock prices decline

CHAPTER 4

Using Forward Contracts to Manage Risk

Firms face price risk in their input markets and their output markets. Price risk exists because the future price of the inputs and outputs are unknown today. Price changes in the input market or output market can adversely affect a firm's bottom line.

Forward contracts are routinely written to help shift price risk on commodities such as crude oil, heating oil, copper, grains, and livestock. However, forward contracts are not limited to commodities. Forward contracts can also help firms protect themselves against adverse changes in interest rates or adverse changes in foreign exchange rates.

Forward contracts provide firms with an efficient means to manage price risk. The main advantage of forward contracts is their flexibility. Forward contracts can be structured for nearly any firm-specific situation that requires protection against adverse price changes. This is a terrific advantage. A disadvantage of a forward contract, however, is that the firm is bound by the terms of the forward contract even when prices move advantageously. Another disadvantage is the risk that the counterparty to a forward contract will default. That is, the counterparty may not live up to the terms of the contract.

In this chapter, we illustrate how forward contracts are used to shift risk. We start by showing how commodity price risk can be managed. Then we demonstrate how forward contracts can be used to manage interest rate risk and foreign exchange risk. Much of the material in this chapter also describes how futures contracts can be used manage risk. Only the discussion of forward rate agreements (FRAs; see Section 4.2) and their use in managing interest rate risk is specific to forward contracts.

4.1 Using Forwards to Manage Commodity Price Risk

4.1.1 Buying Forwards to Hedge Against Price Increases

The user of a raw material faces the risk that the price of that commodity will rise. For example, many industrial firms require crude oil as an input to their production processes. If the price of crude oil rises, then these firms' costs will rise. All else equal, this will lower profits and lower firm value. An elementary income statement is:

$$\begin{aligned} &\text{Revenues} = \text{output price} \times \text{units sold} \\ &\underline{\text{Costs}\ \ } = \text{input prices} \times \text{input units purchased} \\ &\text{Profits} \end{aligned}$$

Unless the firm can pass the higher costs on to consumers in the form of higher output prices, its profits will be eroded by higher input (crude oil) prices, and this will cause a drop in firm value. Figure 4.1 illustrates the input price risk faced by a user of a raw material.

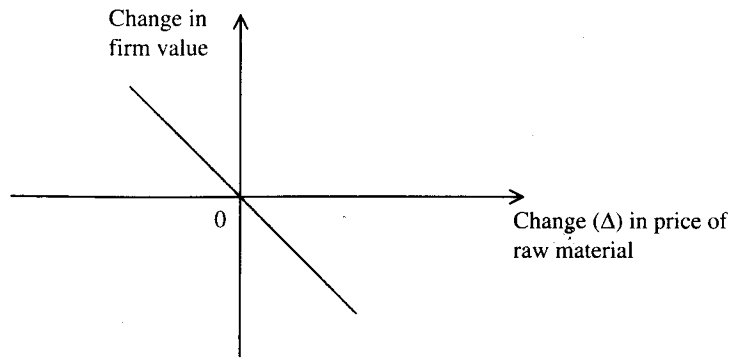

Figure 4.1 An increase in the price of a raw material used by the firm leads to a decline in the value of that firm, all else equal.

A firm facing the risk that the price of one of its inputs will rise can use forward contracts to manage that risk. Buying forward contracts on the raw material will lock in its purchase price. One way to think about using forwards (and futures and swaps) to hedge price risk is that a firm should act to eliminate the price risk in the forward market in the same way that it otherwise would have acted in the spot market. In this case, buying the raw material in the spot market would have eliminated its exposure to the risk that its price would rise. Instead, buying the raw material in the forward market (going long forward contracts) accomplishes the same result; instead of actually buying the raw material today, the firm is committing itself to buy it at a later date. The advantages of buying the forward contract are as follows.

1. There is no cost to taking on a long position in a forward contract; buying the commodity in the spot market requires an initial cash outlay. Because there is an opportunity cost to any cash outlay, it is cheaper to use the forward contract.[1]
2. Buying the good in the spot market will result in storage costs and insurance costs. The firm does not have to incur the expense of storing and insuring the commodity if it buys the forward contract.
3. Should the firm later change its mind about hedging its price risk, or later discover that it has no need for the commodity, it may be easier to offset a long forward position than to find a buyer for a commodity already bought.
4. The amount exposed to default risk in a forward contract is only a fraction of what is at risk in a cash transaction. The principal amount of a forward contract is not at risk; essentially, only the difference between the contracted forward price and the current spot price, times the principal amount, is at risk.
5. The transactions costs (commissions and bid–ask spreads) may be lower in the forward market than in the spot market.

There are also some disadvantages to using forward contracts to manage risk compared with just using the spot market:

1. Two sets of transactions costs may be incurred.
2. The forward market is more or less reserved only for larger organizations.

3. Each party to a forward contract must be concerned about default risk.
4. Unless the asset underlying the forward contract is identical to the item being hedged, the hedge will likely be imperfect.

Figure 4.2 presents the profit diagram for a long forward contract. If the price on the delivery day is greater than the forward price that was agreed upon on the origination day, then the party that is long the forward contract realizes a profit. Suppose a firm is exposed to the risk of an increase in the price of a raw material. This firm then buys a forward contract that constitutes an obligation to buy that raw material in the future. Figure 4.3 show how the exposure to price risk is removed.[2]

4.1.2 Selling Forwards to Hedge Against Price Declines

A producer of a commodity may be exposed to the risk that the price of its output will decline. That is, the firm might be engaged in copper mining, crude oil production, or growing cash crops.

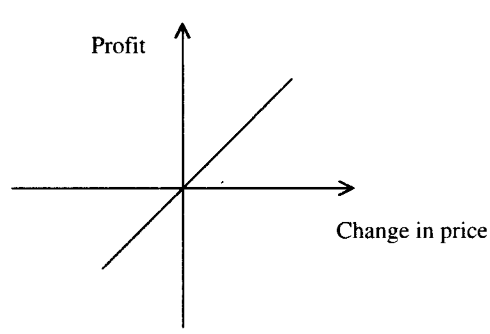

Figure 4.2 Profit diagram for a long forward position. A rise in the price of the good makes a long position in a forward contract profitable.

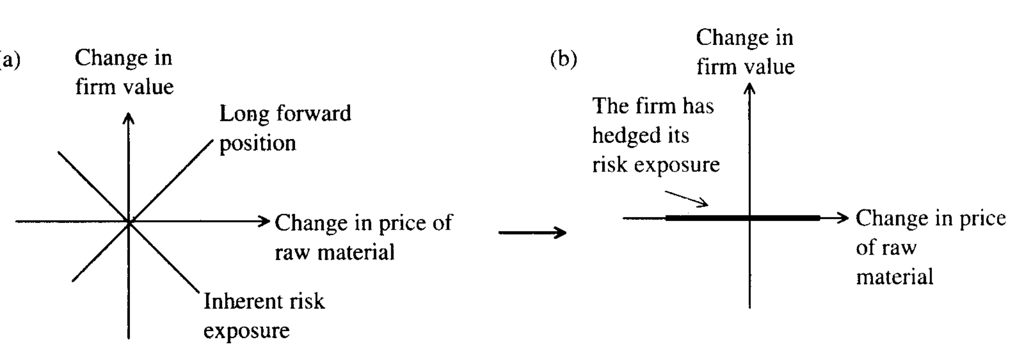

Figure 4.3 A firm that is exposed to the risk that the price of a commodity will rise can manage its risk exposure by buying a forward contract on that commodity (*a*). If the hedge is effective, then all of its risk exposure is removed, as shown in (*b*).

As explained shortly, if the price of the output product declines, and if the product is price inelastic, then the firm's revenues will decline. Figure 4.4 illustrates that when the price of its output declines, the value of a firm declines, all else equal.

An important assumption in the analysis of the effect of a decline in the price of this firm's output is that the demand for its product is price *inelastic*. The "law of demand" states that quantity demanded increases when prices fall. Thus, all else equal, the firm should experience an increase in units sold when the price of its output declines. If the product is price inelastic, then the percentage increase in units sold is less than the percentage fall in its price, so that revenues will decrease. Put another way, when a good exhibits inelastic demand, an $x\%$ decline in its price results in a percentage increase in the quantity demanded that is smaller than $x\%$:

$$\text{Inelastic demand: } \left|\frac{\%\Delta \text{ quantity demanded}}{\%\Delta \text{ price}}\right| < 1$$

For example, if the price of oil rises by 10% and demand for the output of an oil producer declines only 1%, we conclude that the demand for the product is inelastic. If demand for the firm's output good is price inelastic, then all else equal, the decline in price leads to a decline in revenues and a decline in profits:

$$\begin{array}{l} \text{Revenues} = \text{output price} \times \text{quantity sold} \\ \underline{-\text{Costs}} \\ \text{Profits} \end{array}$$

In contrast, if demand for a product is price *elastic,* then changes in price will lead to larger changes in the quantity demanded:

$$\text{Elastic demand: } \left|\frac{\%\Delta \text{ quantity demand}}{\%\Delta \text{ price}}\right| > 1$$

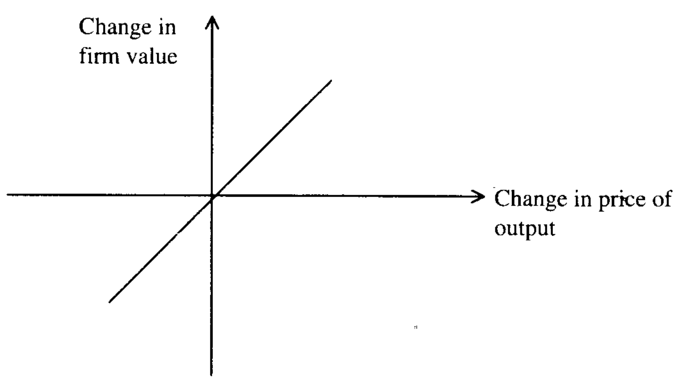

Figure 4.4 When the price of a firm's output declines, its revenues decline (all else equal), and firm value declines.

If demand for firm's product is elastic, it will not be concerned with declining prices because the quantity demanded will increase by such a large amount that its revenues will increase. Figure 4.5 illustrates the difference between elastic demand and inelastic demand.

In general, we should conclude that for an individual firm, demand for its product is price inelastic. Therefore price declines will diminish revenues.[3]

A firm exposed to the risk that the price of a commodity will decline can hedge the risk by selling that good in the spot market today. At times, however, selling the good in the spot market is impossible or not feasible. For example, a wheat farmer who has just planted a crop is exposed to the risk that the price of wheat will decline. Obviously, he cannot sell the wheat in the spot market, since the crop has not been harvested. Instead, the farmer can transact in the forward market in the same way that he would otherwise act in the spot market. That is, he can sell his wheat in the forward market to protect himself. Going short a forward contract, which obligates him to sell the wheat at the contractual forward price, eliminates his exposure to price risk.

Figure 4.6 presents the profit diagram for a firm that is short a forward contract. It shows that a price decline (i.e., the spot price on the delivery day is below the origination day forward price)

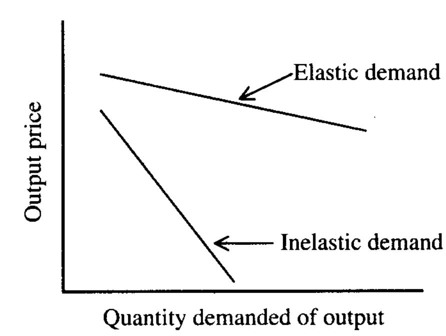

Figure 4.5 When demand is inelastic, a rise in price has only a small impact on the quantity demanded. Elastic demand means that a rise in price leads to a large decline in demand.

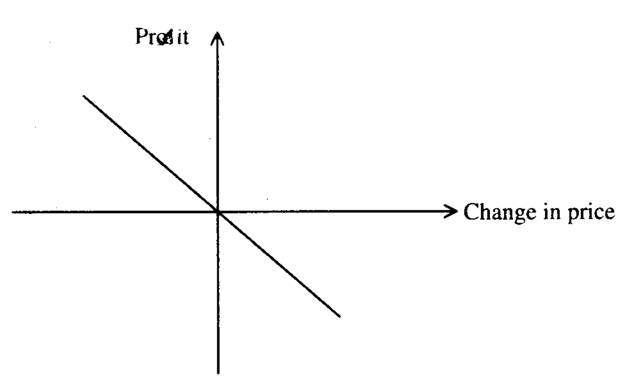

Figure 4.6 Profit diagram for a firm that is short a forward contract. A decline in the price of the good creates a profit for the firm that is short a forward contract.

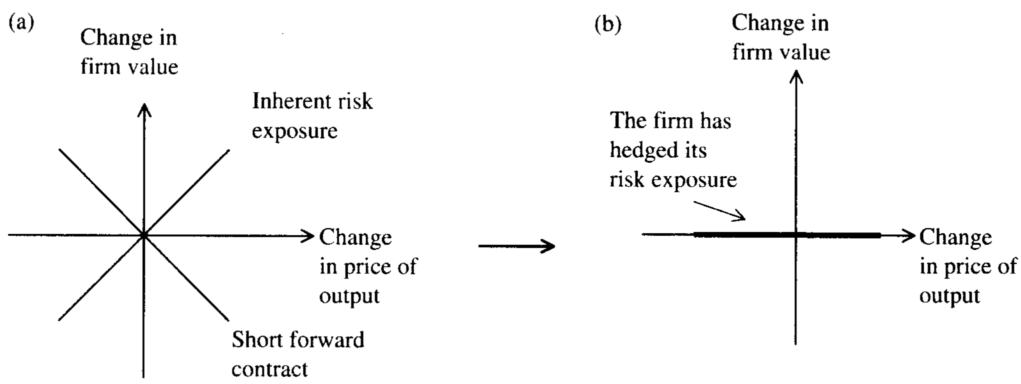

Figure 4.7 A firm that is exposed to the risk that the price of a commodity will fall manages its risk exposure by selling a forward contract on that commodity (*a*). If the hedge is effective, then all of its risk exposure is removed, as shown in (*b*).

leads to profits for a firm that has sold a forward contract. Figure 4.7 combines Figures 4.5 and 4.6 to illustrate that a firm that is exposed to the risk of a price decline can sell a forward contract to hedge its risk exposure.

4.2 Using Forwards to Manage Interest Rate Risk

Recall that a forward rate agreement (FRA) is a forward contract on an interest rate. That is, if interest rates rise above the forward rate, the buyer of the FRA profits. If the interest rate at maturity is below the forward rate that was originally agreed to in the FRA, then the seller of the FRA realizes a profit. The settlement amount is essentially the present value of the lost/gained interest on the notional principal amount [see Equation (3.1)]. Thus, FRAs can be used to hedge against higher or lower than expected interest expense or revenue. It is important to note that a FRA is used to hedge just *one period* of interest.

4.2.1 Hedging Against an Increase in Interest Rates

A firm that buys a FRA does so to hedge against the risk that interest rates will rise. If interest rates subsequently rise, the hedger will lose money in the spot market and make an offsetting profit with the FRA it bought. For example, a firm that is a borrower at a floating interest rate faces the risk that interest rates will rise; a higher interest rate will lead to an increase in the firm's interest expense, and lower profits, all else equal. When a firm issues a floating-rate debt instrument, it is borrowing at a floating rate, and it loses when interest rates rise.[4] Another firm that plans to issue short-term debt some time in the near future may wish to lock in its interest expense today. Investors who own fixed-rate debt will experience a decline in the value of their assets if interest rates rise. For all these cases, buying a FRA serves to hedge the unwanted effect of higher interest rates.

Figure 4.8 shows the risk exposure of a firm that fears rising interest rates. Figure 4.9 illustrates the profit diagram for a long FRA position. Combining the two creates a hedge.

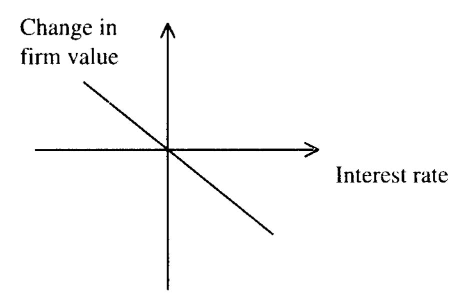

Figure 4.8 Risk exposure of a firm that fears rising interest rates.

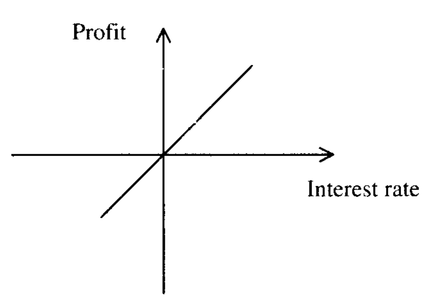

Figure 4.9 Profit diagram for a long FRA position.

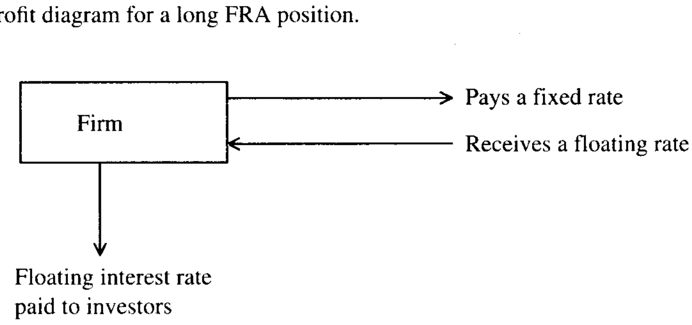

Figure 4.10 A FRA is equivalent to a one-period interest rate swap. It converts a floating-rate expense into a fixed-rate expense for a firm that is exposed to the risk that interest rates rise, such as a firm that has a floating-rate bond outstanding.

Another vehicle for visualizing what entering into a long FRA contract accomplishes in the way of interest rate risk management is shown in Figure 4.10. The firm is paying out a floating interest rate to investors. A long position in a FRA is equivalent to a one-period interest rate swap in which the firm is the fixed-rate payer, and it receives the floating rate. If the formula that establishes both floating rates is the same, then the firm is left with a locked-in fixed rate as a future expense.

Suppose that a firm has borrowed \$60 million at a floating rate. The floating rate is reset every 6 months, and 6-month LIBOR at time t establishes the interest payment on the loan at time $t+1$. Thus, the firm knows what its next interest expense will be, 6 months from today. A FRA can be used to hedge one floating-rate interest rate payment, to be made at one future date. To hedge the interest expense it faces a year hence, the firm will buy a 6×12 FRA with a notional principal of $60 million. Suppose the price of the FRA is 8%. Table 4.1 illustrates how the firm has hedged itself.

Note in Table 4.1 that the profit/loss on the FRA is realized six months hence, while the actual interest expense on the loan is paid out 12 months hence. The interest payment to be made six months hence is already known, because LIBOR at time zero determines the interest payment on the floating rate note six months hence.

To be made comparable, and to illustrate the results of the hedge more clearly, we can compound the FRA's profits or losses so that they can be subtracted from or added to the actual interest expense on the floating-rate note. Table 4.2 performs this step. A perfect hedge is not

TABLE 4.1 Hedging Interest Rate Risk with a Long FRA Position: An Example

Six Month LIBOR, Six Months Hence	Interest Expense on the $60 million loan, paid Twelve Months from Today	Profit (+), or Loss (−) on the FRA, realized Six Months from Today[1]
7.0%	(0.070/2) $60 million = $2.10 million	−$291,410.63
7.5%	(0.075/2) $60 million = $2.25 million	−$145,352.34
8.0%	(0.080/2) $60 million = $2.40 million	$0
8.5%	(0.085/2) $60 million = $2.55 million	+$144,651.49
9.0%	(0.090/2) $60 million = $2.70 million	+$288,607.19

[1]The figures in this column are computed by means of Equation (3.1); it is assumed that there are 181 days until settlement and the day count method is actual/360. Equation (3.1) is repeated here:

$$\left|\frac{P[r(t1,t2)-fr(0,t1,t2)](D/B)}{1+[r(t1,t2)(D/B)]}\right|$$

For example, the $291,410.63 loss on the FRA realized when six-month LIBOR is 7% is found by calculating:

$$\left|\frac{\$60,000,000[0.07-0.08](181/360)}{1+[0.07(181/360)]}\right| = \$291,410.63$$

TABLE 4.2 Computing Profits or Losses for a FRA

Six-Month LIBOR, Six Months Hence	Interest Expense, Paid *Twelve* Months from Today	*Compounded* Profit (+), or Loss (−) on the FRA[1]	Total Effective Interest Expense
7.0%	$2.10 million	−$301,666.67	$2,401,666.67
7.5%	$2.25 million	−$150,833.33	$2,400,833.33
8.0%	$2.40 million	$0	$2,400,000.00
8.5%	$2.55 million	+$150,833.33	$2,399,166.67
9.0%	$2.70 million	+$301,666.67	$2,398,333.33

[1]For example, the compounded loss if LIBOR is 7% is computed as follows:

$$\$291,410.63[1+0.07(181/360)] = \$301,666.67$$

achieved (note the figures in the last column) because the FRA uses an actual/360 day count method, while we assumed that the loan uses a 30/360 day count method.

4.2.2 Hedging Against a Decline in Interest Rates

Sellers of FRAs wish to hedge against the risk that interest rates will fall. A decline in interest rates will reduce interest income for a floating-rate lender. But if that lender has sold a FRA, the lender will have hedged, since the drop in interest rates will also result in a profit on the FRA. An investor who owns a portfolio of short-term debt instruments or floating-rate bonds is also exposed to a decline in interest rates. Banks and other financial institutions that have made floating-rate loans, and financed them with fixed-rate sources of capital, also fear declining interest rates; their interest expense is fixed, but their interest income from the loans is variable.

Figure 4.11 illustrates the risk exposure for a firm that fears a decline in interest rates. The decline will lead to lower interest income, hence lower profits and a drop in firm value. Figure 4.12 is the profit diagram for a short FRA position. Selling a FRA will be profitable if interest rates fall below the contracted forward rate; the profits will help offset the decline in the value of a firm that depends on interest income from short-term securities or floating-rate notes. Instead of paying interest expense, this firm will receive interest income. Because the profit or loss on the FRA is reversed when the firm is short the FRA, the firm has greatly reduced the dispersion of interest income.

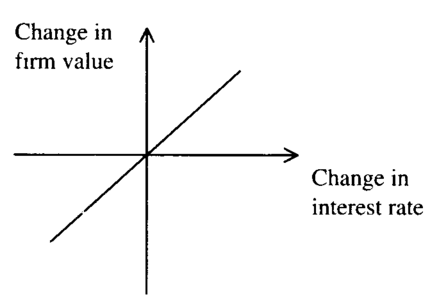

Figure 4.11 Risk exposure of a firm that fears falling interest rates.

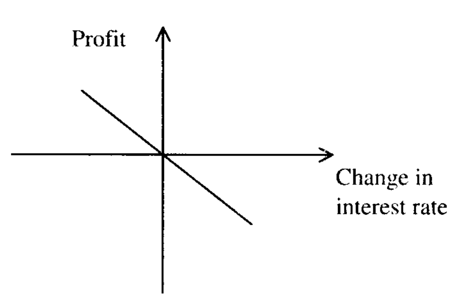

Figure 4.12 Profit diagram for a short FRA position; profits are made if interest rates fall.

4.3 Using Forward Foreign Exchange Contracts to Manage Risk

Firms that buy and/or sell products in foreign countries are exposed to risks that the prices of foreign currencies will change. Even if a company does not import or export goods, it may be impacted by changing exchange rates if its competitors are either foreign-based exporters or domestic firms that import raw materials. In this section, we discuss the nature of this exposure to exchange rate risk, both from an income statement perspective and from a balance sheet perspective.

Understanding the jargon is important. If the home country is Great Britain, then the price of a foreign currency, say yen (¥), is expressed as £/¥. This is the British pound price of a Japanese yen. If this exchange rate rises, then we say that the yen has risen in value, or that the yen has strengthened (relative to the pound). If the yen becomes more expensive, then it is equivalent to saying that the pound has dropped in value, relative to the yen.

4.3.1 Managing the Risk That the Price of a Foreign Currency Will Rise

Consider a U.S. firm that imports its raw materials and must pay for them in a foreign currency. Assume that this U.S. firm is importing goods from Japan and thus must pay yen for its raw materials, which represents a cost. The U.S. firm wishes to maximize its dollar-denominated profits. A simple income statement is:

$$\begin{array}{ll} \text{Revenues} & = \$\text{Revs} \\ \underline{-\text{Costs (in yen)} \times \$/\text{¥}} & \underline{= \$\text{Costs}} \\ \text{Profits (in dollars)} & \end{array}$$

The $/¥ exchange rate is what converts the firm's yen-denominated expenses into dollar expenses. This firm is exposed to the risk that the $/¥ exchange rate will rise. If it costs more dollars to buy a yen, then all else equal, this importer's dollar-denominated expenses will rise. Profits will fall and firm value will decline. We are assuming that the firm is unable to pass its increased costs on to the buyers of its products.

Another scenario is based on the balance sheet. A simple balance sheet has the following headings:

Assets	**Owners' Equity Liabilities**

Think of this balance sheet in terms of economic values, not accounting figures. If the market value of a firm's liabilities is subtracted from the market value of its assets, we are left with the market value of the stockholders' equity. The goal of a firm is to maximize the value of the owners' common stock, denominated in its home currency.

Suppose that some of the firm's liabilities are denominated in a foreign currency, and their value (in terms of the firm's local currency) is dependent on the exchange rate. It follows that if the exchange rate rises, the value of the liabilities rise. If all the firm's assets are denominated in the local currency, then assets can equal (liabilities+owners' equity) only if the value of owners' equity declines.

For example, suppose that a firm in France wishes to maximize the value of its common stock in terms of euros. This French firm has some of its liabilities with values that are denominated in terms of yen. The liabilities may be bonds that were issued in Japan, with coupon interest and principal denominated in yen, or perhaps some yen-denominated payables. This French firm has no yen-denominated assets. Using some specific figures, we might have the following balance sheet, in millions of euros, and assuming an initial exchange rate of €0.01 for each yen (€0.01/¥).

Assets	**Liabilities and Owners' Equity**	
Euro denominated = €145	Euro denominated liabilities = €50	
	Yen denominated liabilities = ¥3000	= €30
	Owners' equity	= €65
Total assets = €145	Total liabilities and owners' equity	= €145

Now suppose that the exchange rate rises from €0.01/¥ to €0.012/¥. The assets and liabilities that are denominated in euros remain unchanged in value, all else equal. But the euro value of the yen-denominated liabilities rises to €36 million. Because the stockholders' claims are a residual, the value of the firm's common stock must decline to €59 million. The increase in the value of the firm's liabilities is accompanied by a commensurate decline in the euro value of owner's equity.

Thus, we conclude that a firm with substantial liabilities whose values are denominated in terms of a foreign currency is exposed to the risk that the exchange rate (home currency/foreign currency) will rise.

For this reason, many firms try to match their liabilities and assets in terms of currencies. In other words, this firm could hedge itself by selling €30 million of its assets and buying ¥3000 million of yen-denominated assets with the proceeds. By doing this, a rise in the €/¥ exchange rate will increase the value (expressed in euros) of both its assets and its liabilities by the same amount; the value of the firm's common stock will remain unchanged.

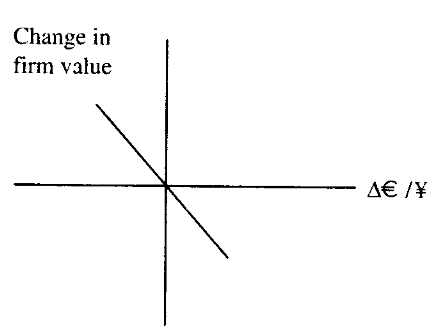

Figure 4.13 A rise in the euro price of a yen will cause a drop in the value of this firm.

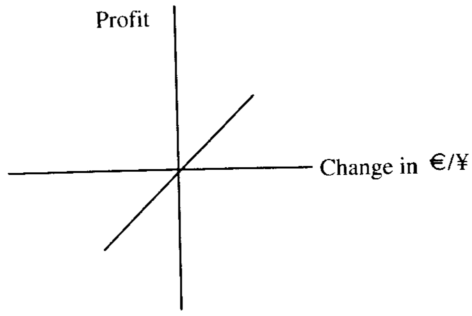

Figure 4.14 Profit diagram for long forward position in yen.

The French firm can also use forward contracts on foreign exchange to manage its risk exposure. Figure 4.13 illustrates the risk exposure of the French firm. Since a rise in the price of the yen causes a decline in the value of its stock, the French firm should buy a forward contract on yen to hedge. Figure 4.14 shows the profit diagram for a long forward position. If the firm depicted in Figure 4.13 buys yen forward, it can be hedged against a rise in the Japanese currency.

4.3.2 Managing the Risk That the Price of a Foreign Currency Will Decline

The exporter of a finished product that is paid in units of a foreign currency is exposed to the risk of a falling exchange rate, where the exchange rate is expressed as home currency/foreign currency. Again, think in terms of a simple income statement. This time consider the example of a Canadian firm that exports its production output to the United States and is paid in U.S. dollars. The firm wishes to maximize its profits in terms of Canadian dollars (Can$):

Revenues (U.S.$) × Can$/U.S.$	= Can$
−Costs	= Can$
Profits	= Can$

The Canadian firm fears that the Can$/U.S.$ exchange rate will decline. If this happens, its revenue stream, denominated in Canadian currency, will decline, and so will its profits. To hedge itself, the Canadian firm should sell U.S. dollars forward. If Can$/U.S.$ falls, the firm's revenues denominated in local currency will decline, but the firm will also realize a profit because it sold U.S. dollars forward.

As before, we can also analyze an exposure to risk in terms of a firm's balance sheet. Suppose that the value of some of a firm's assets is dependent on the exchange rate. It follows that if the exchange rate declines, and if all its liabilities are denominated in the local currency, the lower asset value must be accompanied by a decline in the value of the firm's common stock (denominated in the home currency).

For example, suppose that a firm in France (wishing to maximize the euro-denominated value of its common stock) has some of its assets with a value that is denominated in terms of British pounds. The assets may be British securities (stocks or bonds), or perhaps some undeveloped British real estate, and they are currently worth £30 million. This French firm has no pound-denominated liabilities. Assume an initial exchange rate of €1.5/£, and consider the following simple balance sheet:

Assets (in millions)	Liabilities and Owners' Equity (in millions)
Euro denominated = €100	Euro-denominated liabilities = €80
Pound denominated = £30 = €45	Owners' equity = €65
Total assets = €145	Total liabilities and owners' equity = €145

Now suppose that the exchange rate declines from €1.5/£ to €1.4/£. The assets and liabilities that are denominated in euros remain unchanged, all else equal. But the euro value of the British assets declines to €42 million. Because the stockholders' claims are a residual, the value of the firm's common stock must decline to €62 million.

Thus, we conclude that a firm with substantial assets whose values are denominated in terms of a foreign currency is exposed to the risk that the exchange rate (home currency/foreign currency) will decline.

This French firm could hedge itself by retiring €45 million of its euro-denominated liabilities and then issuing £30 million of pound-denominated debt. Upon doing this, a decline in the €/£ exchange rate will reduce the value (expressed in euros) of some of its assets *and* its liabilities; the value of the firm's common stock will remain unchanged. Alternatively, the firm could hedge by selling £30 million in the forward market.

As another case, consider a U.S.-based mutual fund that believes not only that some Japanese stocks are undervalued (in terms of their yen prices) but that the $/¥ exchange rate is likely to decline. A fall in the yen would offset the anticipated rise in the price of the Japanese stocks. The mutual fund can hedge its initial investment (but not its profit or loss) in the stocks by selling yen forward.

For example, assume that the spot exchange rate is $0.008696/¥. The mutual fund invests $10 million in the Japanese stocks. Therefore, at the current spot rate, it will invest ¥1.15 billion in Japanese stocks. To hedge its exposure to a decline in the yen, the fund sells ¥1.15 billion forward at the current forward price of $0.009076/¥. This forward contract is for delivery one year hence.

One year hence, the mutual fund is pleased to note that it was correct on both accounts. The value (in yen) of its Japanese stocks rose to ¥1.426 billion (up 24% in yen). But at the same time, the spot exchange rate declined to $0.008475/¥. Thus, the fund also made a profit on the forward contract. Its total dollar-denominated profit is computed as follows:

Sell stocks for ¥1.426 billion.

Exchange yen for dollars at the spot exchange rate: ¥1.426 billion × $0.008475/¥ = $12,085,350

Profit on forward contract: ¥1.15 billion ($0.009076/¥ − $0.008475/¥) = $691,150

Total dollar profit: $12,085,350 + $691,150 − $10,000,000 = $2,776,500

Dollar rate of return = ($12,776,500 − $10,000,000)/$10,000,000 = 27.765%

Had the fund not hedged itself, its dollar profit and dollar rate of return would not have been so impressive because of the decline in the price of yen. Even though the Japanese stocks rose in value by 24%, the fund's profits would have been only 20.85% [($12,085,350 − $10,000,000)/ $10,000,000] because the yen dropped in value.

In this example, the fund sold its yen-denominated initial investment forward. It could not sell forward the total amount of yen it subsequently had, one year later, because it did not know how many yen it would have at that time. The value of the Japanese stocks rose, so in this example, the company ended up having more yen to sell. Had the prices of the Japanese stocks declined, it would have had less than ¥1.15 billion to exchange for dollars one year later.

4.4 WHAT QUANTITY SHOULD BE BOUGHT OR SOLD FORWARD?

So far, we have analyzed the direction of a firm's risk exposure and determined whether forward contracts should be bought or sold to reduce the variance of a firm's profits, and/or to reduce the volatility of firm value due to changes in prices. But we have not had much to say about how much should be bought or sold in the forward market to achieve the optimal results.

When an individual transaction is being hedged, it readily follows that the amount to buy or sell forward should equal the amount that underlies that transaction. If a firm will buy 5000 barrels of oil at the end of each of the next four quarters, then it should buy 5000 barrels of oil for forward delivery 3, 6, 9, and 12 months hence. A firm that knows that it will have $30 million in variable-rate debt outstanding for the next 2 years, but wishes to fix its future interest expense will want to buy a strip of FRAs, with delivery dates that correspond to each of the eight future dates on which the interest rate on its debt will be set, and each of which has a principal amount of $30 million.[5] A British firm that has an account receivable in the amount of €30 million will sell that many euros forward, with delivery on or around the date on which it expects to be paid; when it delivers the €30 million to satisfy the terms of the forward contract it will want its counterparty to pay British pounds.

Hedging economic profits (cash flows) or equity value against price risk is more difficult to accomplish. But in Chapter 2 you learned the basic tools that are required to manage risk when a firm has such goals. Suppose that a U.S. exporting firm has decided to hedge the next eight quarterly (2 years) cash flows against foreign exchange price risk; assume that the dollar price of Japanese yen is of concern. The firm must first estimate how its cash flows are related to changes in the $/¥ exchange rate. To do this, it can estimate different regression models with historical data, or use Monte Carlo simulation techniques to evaluate the relationship. Suppose that it estimates that a $0.001 decline in the price of the yen will lead to a decline of $200,000 in quarterly profits (and similarly, quarterly profits will rise at the rate of $200,000 per $0.001 increase in the price of yen). The firm will then want to sell forward a sufficient number of yen to be able to realize a profit of $200,000 on the forward contract for every $0.001 fall in the yen's price. This means that $200,000/($0.001/¥) = ¥200,000,000 should be sold for forward delivery at *each* quarterly delivery date (i.e., eight forward contracts calling for the forward delivery of ¥200 million 3, 6, ..., 21, and 24 months hence).[6] This will lock in the forward prices that exist as of "today." Suppose that today's spot price and forward price for delivering yen 9 months hence is $0.0086/¥. Then, 9 months from today, the actual spot price is $0.0080/¥. The firm's operating profits *should* decline by about $120,000. But the firm will realize a profit equal to ¥200,000,000($0.0086/¥ − $0.0080/¥) = $120,000 on the forward contract.[7] Thus, the change in the price of the yen has a reduced impact on total cash flow for the firm that has hedged itself by selling yen forward.

Similar techniques are needed if the firm is managing equity volatility induced by price volatility. The firm must first estimate how equity value is affected by a change in a price of some good. Then, a sufficient quantity of the good must be bought or sold forward to result in a profit in the forward contract equal to the decline in equity value. The firm must also decide what horizon length is of concern.

Chapter 7 presents other aspects concerning the decision of the amount to be bought or sold forward.

4.5 Summary

Forward contracts are used to manage price risk. Each contract, with its own delivery date, is used to manage a single cash flow. Hedgers buy forward contracts to protect themselves against price increases. They sell forward contracts to hedge against price declines. When the spot price at delivery is greater than the forward price, the party that has a long position in the forward contract profits, and the short loses. The party that sells forward contracts realizes a profit when the delivery day price is below the forward price that was agreed upon when the contract was originated.

Products that create revenues for a firm expose it to the risk that output prices will decline, assuming that demand for the firm's product is price inelastic. Inputs are costs, or expenses, and firms fear that the prices of these goods will rise. In both these cases, profits decline.

When a price decline causes a firm's assets to decline in value, then forward contracts should be sold. If a price rise causes a firm's assets to lose value, then forwards should be bought. Price changes will also affect liability values. If a price decline causes liabilities to increase in value, all else equal, the firm's stockholders will see the value of their position be eroded by the price drop and the firm should then sell forward contracts to hedge this risk. If a price rise leads to an increase in a firm's liability values, all else equal, then a long forward position is needed.

FRAs are used to protect firms against interest rate fluctuations. A party will buy a FRA to hedge against interest rate increases. FRAs will be sold when interest rate declines are feared.

Forward foreign exchange contracts are used to manage the risk that is created by changes in exchange rates. For example, suppose the dollar is the home currency, and *fx* is the foreign currency. Then a U.S.-based exporter that is paid *fx* for its product faces the risk that the $/*fx* rate will decline, because that will lead to decline in its dollar-denominated revenues. Alternatively, a U.S.-based importer that must pay *fx* for its imports faces the risk that the $/*fx* exchange rate will rise. It will take more dollars to buy one unit of *fx*, creating greater dollar-denominated expenses for the importer.

Another model of analyzing exchange rate risk focuses on the balance sheet. A firm with *fx*-denominated assets fears a decline in the value of *fx*. Another firm may have *fx*-denominated liabilities; it faces the risk that the price of *fx* will rise. In either case, the value of the stockholders' position declines.

Finally, in the last section of the chapter, we explained that to determine how much of an asset should be bought or sold forward, the firm should estimate how its cash flow or equity value is affected by price changes. Then, the best hedge dictates the buying or selling forward of a sufficient quantity of the asset to result in profits on the forward contract that equal the decline in profits or the decline in the firm's common stock.

References

Luenberger, David G. 1998. *Investment Science*. New York: Oxford University Press.

Wilson, Richard S. 1997. "Domestic Floating-Rate and Adjustable-Rate Debt Securities," in *The Handbook of Fixed Income Securities*, 5th ed, Frank J. Fabozzi, ed. Burr Ridge, IL: Irwin Professional Publishing.

Notes

[1] But the forward price will often reflect some of this advantage, as well as the benefit described in item #2.

[2] For ***all*** price risk to be removed, it is necessary that (a) the amount of the raw material underlying the forward contract equal the amount it will have to buy in the future to fulfill its production requirements, and (b) the quality of the raw material underlying the forward contract and its delivery location be the same as the quality and delivery location required by the firm in its production process. For example, there are many grades of crude oil, and relative prices can fluctuate. Similarly, the price of crude oil at the physical location of a chemical production plant will likely be different from the price of crude oil at the location at which it is removed from the ground.

[3] We have deliberately ignored another complication. Frequently, a firm's production is negatively correlated with price. Consider a farmer. When his harvest is large, it is likely that the harvests of many farmers are large, and therefore product prices will be lower. In other words, quantity risk naturally serves as a hedge in many cases. See Luenberger (1998, pp. 287–290). We thank Jerald Pinto for directing our attention to this situation.

[4] A floating-rate debt instrument is a security with a coupon or interest rate that periodically changes. The change in the level of interest is typically tied to an index. The index may be a short-term index such as LIBOR, or it may be a long-term index. See Wilson (1997) for a discussion of U.S. floating-rate securities. Floating rate notes are often called FRNs.

[5] If the firm wanted to lock in a fixed interest rate, it would probably prefer to enter into an interest rate swap as the fixed rate payer. Swaps will be introduced in Chapter 11.

[6] A currency swap would accomplish the same goal.

[7] Note two items. First, the firm only *estimated* that its profits would decline by $200,000 if the yen declines by $0.001. The actual change in profits is a random variable. Second, the firm must realize that it is actually hedging against a change in the *forward* price. In other words, profits on the forward exchange contract are realized if the spot price at delivery is less than the original forward price. But changes in operating profits have been estimated as being a function of changes in the spot price of the yen.

PROBLEMS

4.1 What price risk does a gold mining firm typically face? Suppose that demand for gold mining firm A's output is totally independent of the price of gold, while demand for gold mining firm B's output is price inelastic. How will this situation affect how firm A will use forward contracts on gold to protect itself, relative to how firm B will use gold forward contracts?

4.2 A cereal manufacturer buys grains from farmers. What price risk does the cereal manufacturer face? What price risk does the farmer face? How can each party use forward contracts on wheat and corn to manage its price risk?

4.3 Banks and other financial institutions have issued most of the floating-rate debt outstanding in the United States. Why do you think this is so?

4.4 A bank borrows $10 million for 12 months, and lends $10 million for 9 months, both at fixed interest rates. Discuss the interest rate risk that the bank is exposed to. How can it use a FRA to hedge this risk?

4.5 A bank borrows $10 million for 9 months, and lends $10 million for a year, both at fixed interest rates. Discuss the interest rate risk that the bank is exposed to. How can it use a FRA to hedge this risk?

4.6 A firm believes that it will have to borrow money 4 months from today. To what interest rate risk is this firm exposed? If it believes that it will need the funds for 2 months, how can it use a FRA to hedge this risk?

4.7

a. A firm plans on issuing $25 million in 9-month commercial paper. It expects to sell this security 6 months from today. How can it use a FRA to hedge this risk? What is the start and end date of the FRA it should use? That is, should it buy or sell an "*N*"×"*M*" FRA? What are the values of "*N*" and "*M*"?

b. Suppose that the price of the "*N*"×"*M*" FRA is 6%. Prepare a table showing the interest expense on its commercial paper. Assume that the day count method for both the commercial paper and the FRA is 30/360.

Future Relevant Interest Rate	Interest Expense on Issued Commercial Paper	Profit/ Loss on FRA
5%		
5.5%		
6%		
6.5%		
7%		

4.8 A financial institution has a portfolio of floating-rate loans. Its capital structure consists mainly of 5-year CDs and long-term fixed-coupon rate bonds that it sold several years ago. Discuss the interest rate risk that it faces. How can FRAs be used to manage its risk exposure?

4.9

a. A Japanese firm knows that in 4 months it will have 45 million French Francs to deposit in a French bank. It will leave these funds deposited in this interest-earning account (earning FFR-denominated interest) for a period of 8 months, at which time it will need Japanese Yen to pay one of its Japanese suppliers. Discuss the interest rate risk the Japanese firm faces. Discuss the nature of the foreign exchange risk that it faces.

b. Will the Japanese bank want to buy or sell a FRA? Suppose that 4×12 FRAs in France are quoted at 9%. Four months hence, spot 8-month French interest rates are 7.8%. What profit or loss was realized on the FRA?

4.10 A Japanese firm has all its assets in Japan and sells all its products in Japan; all its raw materials are produced domestically, as well. Last year, this firm borrowed money from a U.S. bank. It must pay both interest and principal in U.S. dollars. Discuss the exchange rate risk faced by this firm. How can it use a forward foreign exchange contract to manage this risk?

4.11 A U.S. mutual fund has invested in a portfolio of British stocks. However it does not wish to be exposed to exchange rate risk.

a. Is the fund exposed to the risk that the \$/£ rate will rise or fall?

b. To hedge, should it buy or sell British pounds forward?

c. Suppose that the fund invests \$16 million in British stocks. The spot exchange rate is \$1.68/£. The forward rate for delivery 6 months hence is \$1.673/£. Six months later, the stocks are sold for £9 million, and the spot exchange rate is \$1.70/£. What final profit or loss would the fund have realized, in terms of dollars, had it originally hedged its initial investment in British stocks? What was its dollar-based *annualized* rate of return? What would have been its dollar-denominated profit and rate of return, had it not hedged?

4.12 A Japanese firm imports U.S. apples and pays for them with dollars. What price risks does it face? How can it use forward contracts to hedge its price risks?

4.13 An insurance company owns a coupon bond that pays interest every November 15 and May 15. The face value of the bond is \$45 million, and the coupon rate is 9%. The bond matures on May 15, 2002, and the firm plans on holding the bond until maturity. The insurance company is concerned about the total funds that it will have available on May 15, 2002. Today is October 15, 1997. Discuss how FRAs can be used to manage the insurance company's reinvestment risk (the risk it faces concerning the rates at which the coupons will be able to be reinvested).

4.14 An international portfolio manager located in Germany expects to receive \$60 million to invest in German stocks next month. Discuss the price risks the manager faces. How can a forward foreign exchange contract be used to manage the currency risk exposure?

4.15 A firm uses historical data to estimate the following equation:

$$Y = 10 - 4500X$$

where Y = the change in annual cash flows and X = the change in 1-year LIBOR, in basis points. Fully discuss how the firm can use a FRA to manage its risk exposure regarding next year's cash flow, including a discussion of the principal amount of the FRA that should be used.

4.16 The prospectus to an international mutual fund states that the fund may enter into contracts to purchase or sell foreign currencies at a future date.

a. Under what scenario or situation would it want to buy forward exchange contracts? Under what scenario or situation would it want to sell forward exchange contracts?

b. The fund prospectus states that "such hedging...does not prevent losses if prices of such securities decline. Furthermore, it precludes the opportunity for gain if the value of the hedged currency should rise." Discuss what these statements mean.

4.17 A Swiss fixed-income mutual fund has invested €50 million in long-term British bonds having a coupon rate of 6%. The current exchange rate is €0.84/£.

a. What interest rate risk does the fund face? Why? How can it use a FRA to manage this risk exposure?

b. What exchange rate risk does it face? Why? How can it use a forward exchange contract to manage this risk exposure?

4.18 A Japanese bank expects to lend €10 million to a Spanish firm. If negotiations are successful, the loan will be made 1-month from today. The terms of the loan have already been established: the Spanish firm is to pay a fixed rate of 5%; the term of the loan will be one year; interest and principal, in euros, will be repaid to the bank 1-year after the loan is made. The Japanese bank is interested in maximizing the yen-denominated wealth of its Japanese stockholders.

a. Clearly discuss the nature of the interest rate risk the Japanese bank faces.

b. Clearly state how the Japanese bank can use forward rate agreements to manage the interest rate risk it faces. Explain how the FRA will reduce the interest rate risk faced by the bank. Be as precise and thorough as you can, given the information provided.

c. Should the bank desire that the contract rate on the FRA be above or below 5%? Why?

d. Clearly discuss the nature of the foreign exchange rate risks the bank will face after the loan is actually made.

e. Given your response in part d, clearly state how the bank can use forward exchange contracts to manage the exchange rate risk it will face after the loan is actually made. Again, be as precise and thorough as you can, given the information provided. Explain how the forward exchange contract will reduce the exchange rate risk faced by the bank.

4.19 Which of the following is true?

a. A firm will sell forward contracts to hedge against the risk that the price (cost) of its raw materials will rise.

b. A firm will sell forward contracts to hedge against the price (cost) of its raw materials falling.

c. A firm will sell forward contracts to hedge against the selling price of its output rising.

d. A firm will sell forward contracts to hedge against the selling price of its output falling.

4.20 A French food manufacturer uses Virginian (U.S.) peanuts as a raw material. It pays dollars for peanuts. This French firm

a. fears an increase in the price of a dollar (an increase in the €/$ exchange rate) and an increase in the price of peanuts

b. fears an increase in the price of a dollar and a decrease in the price of peanuts

c. fears a decrease in the price of a dollar and an increase in the price of peanuts

d. fears a decrease in the price of a dollar and a decrease in the price of peanuts

CHAPTER 5

Determining Forward Prices and Futures Prices

This chapter explains the theory of forward pricing. The forward price of a commodity is the price that is quoted today for delivery of the commodity in the future; the price is contracted today but is paid when the good is delivered in the future. Forward exchange rates exist for the forward purchase or sale of foreign exchange. Forward interest rates are rates that exist for future borrowing and lending opportunities.

All the ideas introduced in this chapter also apply to the pricing of futures contracts. In Chapter 6, we explain why futures prices and forward prices might differ. Since, however, the forces that might cause futures prices to differ from forward prices do not seem to be important, we can fairly safely conclude that they ought to be identical.

In theory, forward prices are determined by the force of arbitrage. In other words, if forward prices are "wrong," then arbitrage opportunities will exist. A fundamental assertion in the theory of finance is that well-functioning markets will not permit arbitrage. When arbitrage opportunities exist, markets are not in equilibrium (i.e., supply does not equal demand), and traders will quickly act to exploit arbitrage profits. (An arbitrage profit is riskless, involving a positive cash inflow at one or more dates, and zero cash flows at all dates. In other words, arbitrage requires no investment and no cash outlay. The arbitrageur generates only cash inflows at one or more dates.)

In the first section of this chapter, we discuss the cost-of-carry model for forward commodity prices. Under perfect markets assumptions, this model should determine one forward price. However, markets are imperfect; in particular, the *convenience yield* must be considered when one is determining the theoretical forward prices of commodities. The convenience yield is a unobservable theoretical construct that arises because there are problems in short selling commodities, and those who own the commodities may be reluctant to sell them. Because of this, it turns out that the cost-of-carry model determines only a maximum forward price that precludes arbitrage. Actual forward prices can be less than the theoretically correct forward price.

In contrast, however, this short selling problem does not exist for financial assets such as foreign currencies and interest rates. Thus, ignoring transaction costs, we will be able to compute one single forward price for these items. Theoretical forward exchange rates are computed in Section 5.2 and theoretical forward interest rates in Section 5.3.

5.1 FORWARD COMMODITY PRICES

5.1.1 The Cost-of-Carry Model

Assume that markets are perfect. This means that there are no transactions costs: no commissions or bid–ask spreads. There are no taxes. Market participants can buy or sell goods without affecting prices. There are no impediments to short selling.[1] Traders who short sell get full use of the proceeds. There is no default risk; each of the two parties to every transaction knows that the counterparty will perform as contractually required. Also assume that all individuals are wealth maximizers. This is equivalent to saying that everyone prefers more wealth to less, or that everyone's marginal utility of wealth is positive. Under these assumptions, the basic forward pricing model is:

$$F = S + CC - CR \tag{5.1}$$

forward price = spot price + carry costs − carry return

The forward price F is the theoretical price for forward delivery of one unit of the commodity. The spot price S is the current price per unit of the good in the cash market.

Carry costs are best thought of as the additional costs incurred by buying and holding one unit of the commodity. Interest charges on borrowing to buy the good and the opportunity cost of having cash tied up in the asset are the most prominent of the carry costs. They are the only costs that are relevant in forward exchange and forward interest rate determination. Carry costs for physical commodities would also include the costs of insurance, storage, obsolescence, spoilage, and so on. Typically, we assume that all carry costs are paid at delivery.

There is no carry return for commodities. For financial assets, the carry return consists of the future value of the cash inflows that are provided to the holder. For example, dividend payments on stocks (and the interest earned on any dividends received prior to the forward contract's delivery date) would represent the carry return for a forward contract on common stock. Actual or accrued coupon income on bonds (and any interest that can be earned on coupons received) represents carry return for forwards on bonds. The interest that can be earned on a foreign currency is a carry return for forward exchange contracts. CR is the future value of these benefits of owning the cash asset.

An alternative interpretation of the cost-of-carry model is to view the purchase of a forward contract as a substitute for the actual purchase of the underlying asset in the cash market. If the forward price is correct, an investor should be indifferent between the two methods (forward purchase or spot purchase) of buying the asset. Consider the choice between buying 100 oz. of gold, or going long one gold forward contract. Buying the actual commodity requires a cash outlay. Moreover, there is an opportunity cost to that cash outlay: either interest is lost on the dollar amount if the money is withdrawn from an interest-earning asset, or else funds must be borrowed and interest paid on the principal. This makes the long forward position more desirable. To make the two choices equivalent, the forward price must be higher by the amount of interest that is saved by buying forward. Thus $F = S + CC$. When buying spot gold we must also account for any costs of insuring and storing physical gold between today and the delivery date; the forward price must be even higher to reflect those costs. If the cash asset supplies its owner with benefits such as dividends or interest income, then to reflect the benefits of owning the cash asset, the forward price will be lower.

5.1.2 Proof of the Cost-of-Carry Model

As with all proofs of propositions that are based on the absence of arbitrage, the proof of the forward pricing model will first ask what if forward prices are too high (more than what the theoretical pricing model specifies) and then analyze what will happen if forward prices are too low (less that what the model specifies).

Proof: What if $F > S + CC - CR$?

Then $+F - S - CC + CR > 0$

Next, we will trade so that the cash flows are generated in the directions contained in this inequality. For example, to create a "$-S$", the underlying spot asset must be purchased; "$-S$" is a cash outflow (the minus sign) of S dollars. To create a "$+F$," the forward contract is sold (eventhough there is really no cash flow until delivery).

Today, time 0

Sell forward at F_0.	No cash flow. As explained in Chapter 3, buying or selling a forward contract at its equilibrium price is costless.
Buy deliverable spot asset	$-S_0$
Borrow	$+S_0$
Cash flow at time 0:	0

On the delivery date, time T

Make delivery to satisfy the terms of the forward contract	$+F_0$
Pay back loan principal and interest[2]	$-S_0 - CC = -S_0 - \text{int} = -S_0[1 + h(0,T)]$
Receive any carry return[3]	$+CR$
	$+F_0 - S_0 - CC + CR > 0$

Thus, if F exceeds $S + CC - CR$, an arbitrage profit can be realized.

The steps one takes to arbitrage if $F > S + CC - CR$ is frequently called a **cash-and-carry arbitrage**. In a cash-and-carry arbitrage, the arbitrageur borrows to buy the spot asset, sells a forward contract, and carries the deliverable asset until the forward delivery date.

The set of trades that an arbitrageur takes to exploit the situation when forward prices are too low, $F < S + CC - CR$, is frequently called a **reverse cash-and-carry arbitrage**. In a reverse cash-and-carry arbitrage, the spot good is sold short, the proceeds of the short sale are lent, and a long position in a forward contract is taken.

Proof: What if $F < S + CC - CR$?

Then $F - S - CC + CR < 0$, or

$-F + S + CC - CR > 0$.

Today, time 0

Buy forward at F_0.	0
Sell deliverable spot asset	$+S_0$
Lend the proceeds from the short sale of the spot asset	$-S_0$
	0

On the delivery date, time T

Take delivery to satisfy the terms of the forward contract, and pay the contractually agreed-upon price	$-F_0$

Receive loan principal and interest	$+S_0+\text{int}=+S_0+CC=S_0[1+h(0,T)]$
Pay any carry return to the person to whom you sold the asset	$-CR$
	$-F_0+S_0+CC-CR>0$

In this part of the proof, we assume that the short seller receives full use of the proceeds of the short sale.[4] However, if the forward price is below its theoretical value, any individual who has a long position in the deliverable cash asset could *quasi-arbitrage* by selling it out of her portfolio, thereby receiving the full proceeds from the sale, and then performing all the steps just described. On the delivery date, this individual will be better off than had she continued to hold the cash asset. Thus, the cost-of-carry formula will determine forward prices when short selling is costly as long as some individuals have the deliverable asset in inventory and do not plan to consume it between time 0 and time T.

5.1.3 Examples

5.1.3.1 An Example: No Carry Return

Suppose the spot price of gold is \$280/oz., the gold forward price for delivery six months hence is \$300/oz., and the yearly interest rate is 10%. The theoretical forward pricing model states that the forward gold price should be \$294/oz.:

$$F = S + CC = S[1 + h(0, T)]$$

$$F = 280(1.05) = 294$$

Because $F>S+CC$, borrow to buy an ounce of gold, and sell one overpriced gold forward contract for delivery six months from today:

Today

Sell one forward contract	No cash flow
Buy 1 oz. of gold	−280
Borrow (for six months at 10% annual interest)	+280
	0

At delivery, with gold at	$F_T=S_T=270$	$F_T=S_T=300$	$F_T=S_T=330$
Deliver 1 oz. of gold, fulfilling the contract's terms	+300	+300	+300
Pay back loan with interest	−294	−294	−294
	+6	+6	+6

Thus, regardless of the delivery day price of gold, the arbitrage profit is \$6 per ounce of gold. Note that the profit is independent of the final spot price of gold on the delivery day.

The FinancialCAD function, aaCDF can be used to compute the forward price of a commodity, as shown in Figure 5.1. The slight difference between the FinancialCAD output forward price of 293.9616439 and the price of 294 we just computed is due to the use of the day count method[5] for unannualizing the annual interest rate of 10%. In this example, the FinancialCAD function uses a periodic rate of (182/365)(10%) = 4.9863% because it uses an actual/365 day count method.

aaCDF		
spot price per unit of underlying commodity	280	
rate - simple interest	0.1	
value (settlement) date	24-Sep-2000	
expiry date	25-Mar-2001	
accrual method	1	actual/365 (fixed)
storage cost	0	
convenience value	0	
statistic	2	fair value
fair value	293.9616438	

Figure 5.1 FinancialCAD function aaCDF computes the theoretical forward price of a commodity. Note the two dates and the day count method (accrual method).

aaCDF		
spot price per unit of underlying commodity	280	
rate - simple interest	0.1	
value (settlement) date	24-Sep-2000	
expiry date	23-Mar-2001	
accrual method	2	actual/360
storage cost	0	
convenience value	0	
statistic	2	fair value
fair value	294	

aaAccrual_days		
effective date	24-Sep-2000	
terminating date	23-Mar-2001	
accrual method	2	actual/360
number of business days from an effective date to a terminating date	180	

Figure 5.2 FinancialCAD function aaCDF computes the theoretical forward price of a commodity. Note the two dates and the day count method (accrual method).

In our earlier example, we implicitly used a 30/360 day count method, which produces a six-month interest rate of $h(0, \frac{1}{2}\text{ yr}) = 5\%$.

In contrast, Figure 5.2 solves the same problem, except that the expiry date (delivery date) is March 23, 2001, and the day count method is actual/360. The expiry date is chosen so that delivery occurs 180 days after the contract initiation date; now, delivery takes place a half-year in the future, and $h(0, \frac{1}{2}\text{yr}) = 5\%$. Thus, the theoretical forward price is 294. The FinancialCAD function, aaAccrual_days is used to compute the number of days until delivery.

5.1.3.2 An Example: The Underlying Asset has a Carry Return

Suppose that a forward contract on a share of General Mills stock exists. The delivery day is five months hence. Today's stock price is $36.375. The stock will trade ex-dividend in the amount of $0.275/share two months hence. The interest rate is 6% for all maturities. Compute the theoretical forward price.

The theoretical price is found by using the cash-and-carry pricing model:

$$F = S + CC - CR$$

$$F = 36.375 + (36.375)(0.06)\left(\frac{5}{12}\right) - (0.275)\left[1 + \frac{(0.06)(3)}{12}\right]$$

$$F = 36.375 + 0.909375 - 0.279125 = 37.00525$$

Note in particular how the **future value** of the dividend is subtracted from $S + CC$. The dividend is paid two months hence. It will earn interest over the subsequent **three** months, until the delivery date. Figure 5.3 shows how the FinancialCAD function aaEqty_fwd solves this problem.

5.1.4 Transactions Costs

If there are transactions costs, there is no single theoretically correct forward price. Instead, the forward price will lie between a lower and upper bound. The cost-of-carry formula becomes:

$$S(\text{bid}) + CC1 - CR - T1 \leq F \leq S(\text{asked}) + CC2 - CR + T2 \tag{5.2}$$

aaEqty_fwd		
value (settlement) date	24-Sep-2000	
expiry date	24-Feb-2001	
accrual method	4	**30/360**
rate - simple interest	0.06	
cash price of the underlying equity index	36.375	
statistic	2	**fair value of forward or futures**
dividend payment table	t_14	

t_14
dividend payment table

all the dividend dates from the value date to the expiry date	dividend amount
24-Nov-2000	0.275

fair value of forward or futures	**37.00525**

Figure 5.3 Using aaEqty-fwd to find the theoretical forward price when there is a carry return.

where

$T1$ = transactions costs from the reverse cash-and-carry arbitrage trades of selling the spot good at the bid price, lending the proceeds and buying a forward contract, $T1$ is paid at delivery

$T2$ = transactions costs from the cash-and-carry arbitrage trades of borrowing to buy the spot good at the asked price and selling a forward contract, $T2$ is paid at delivery

$CC1$ = carry costs from the reverse cash-and-carry arbitrage, which should incorporate the arbitrageur's lending rate and, if needed, should be adjusted to reflect the possibility that the short seller will *not* earn the full amount of interest on the proceeds from the short sale

$CC2$ = the carry costs from the cash-and-carry arbitrage trades, which should utilize the arbitrageur's borrowing rate

The lower bound (left-hand side of the inequality) is set by the reverse cash-and-carry arbitrage trades. If the lower bound is violated, buy the cheap forward, sell the spot good, and lend the proceeds.

The upper bound (right-hand side of the inequality) is set by the cash-and-carry arbitrage trades. If the forward price is above the upper bound, sell the overpriced forward, borrow, and buy the spot good.

EXAMPLE 5.1 Consider the example from Section 5.1.3.1 in which the spot price of gold is \$280/oz. and the yearly interest rate is 10%. The forward contract calls for the delivery of 100 oz. of gold. Also assume that the total transaction cost of buying or selling 100 oz. of spot gold, and trading one forward contract, is \$1/oz. There is no carry return. The range of possible forward prices per ounce of gold that precludes arbitrage is then

$$S + CC1 - T1 \leq F \leq S + CC2 + T2$$
$$280 + 14 - 1 \leq F \leq 280 + 14 + 1$$
$$293 \leq F \leq 295$$

The forward price itself can lie between \$293 ≤ F ≤ \$295 per ounce, for a forward contract on 100 oz. of gold. Any forward price below \$293/oz. will allow reverse cash-and-carry arbitrage. If the forward price is above \$295/oz., then cash-and-carry arbitrage is possible.

Alternatively, suppose that the bid price for spot gold is \$279.50/oz and that the asked price for spot gold is \$280.50/oz. In setting the lower bound, spot gold is sold at the bid price. In setting the upper bound, gold is bought at the asked price. Therefore, the no-arbitrage range of forward prices, in dollars per ounce, is

$$279.50(1.05) - 1 \leq F \leq 280.50(1.05) + 1$$

or,

$$292.475 \leq F \leq 295.525$$

Note that transactions costs widen the no-arbitrage bounds.

Next, suppose that in addition, the individual who is lending *you* the gold to short sell will allow you to use only 60% of the proceeds of the short sale. This means that *your* relevant carry costs (interest earned on the short sale of the spot asset) on the reverse cash-and-carry arbitrage (left-hand side of the inequality) is only [(279.50)(0.6)(0.05) =] $8.385/oz. You can only earn $8.385/oz. if you sell gold short and lend the proceeds that you will receive. The range of no-arbitrage forward prices for *you*, for a contract on 100 oz. of gold, is

$$279.5 + 8.385 - 1 \leq F \leq 295.525$$

$$286.885 \leq F \leq 295.525$$

You are at a disadvantage for performing the reverse cash-and-carry arbitrage trades relative to a quasi-arbitrageur who already has the gold in storage and therefore would receive the full proceeds from selling this asset. A quasi-arbitrageur will arbitrage if F falls below 292.475, but *you* cannot arbitrage unless F declines below 286.885/oz.

Example 5.1 illustrates that different individuals will have the ability to arbitrage at different forward prices, depending on their borrowing and lending rates and on the levels of transactions costs they face.

5.1.5 Implied Repo Rates and Implied Reverse Repo Rates

The **implied repo rate** is the rate of return earned on the cash-and-carry trades. In other words, it is the rate of return that a trader earns from buying the deliverable spot asset and simultaneously selling a forward contract.[6] The implied repo rate is effectively a lending rate. The trader buys the spot asset at time 0 for S_0. This requires a cash outflow, just as if money was lent. The sale of the forward contract guarantees a future cash inflow of F at time T. The implied repo rate is the lending rate from performing these trades, from time 0 to time T. It can be found by solving for $h(0,T)$ in the cost-of-carry formula. If there are no carry returns, then

$$F_0 = S_0[1 + h(0, T)]$$

Solving for $h(0,T)$ yields the *unannualized* implied repo rate when carry returns do not exist:

$$\frac{F_0 - S_0}{S_0} \tag{5.3}$$

This periodic, or unannualized implied repo rate can be annualized by multiplying it by $(365/d)$, where d is the number of days between time 0 and time T. Arbitrageurs will compare the implied repo rate on forward contracts to their own borrowing rate. If they can borrow money at a rate below the implied repo rate (which is a riskless lending rate), then there is a cash-and-carry arbitrage opportunity.

If there are carry returns, the cost-of-carry formula can be used to solve for $h(0,T)$ to compute *IRR*, the unannualized implied repo rate:

$$IRR = \frac{F_0 + CR - S_0}{S_0} \tag{5.4}$$

EXAMPLE 5.2 You spend \$582,500 to buy 10,000 shares of a common stock (at \$58.25/share), and you sell a forward contract on these shares at a forward price of \$58.50/share. The delivery date is three months hence. The future value of dividends that will be received on the shares during the next three months is \$4078; this includes dividends received and the interest that can be earned on those dividends. Thus, the implied repo rate is:

$$\frac{(58.50)(10,000) + 4078 - 582,500}{582,500} = 1.129\%$$

The unannualized implied repo rate is 1.129%. The annualized implied repo rate is 4.517% (0.01129×4). If you can borrow at a rate below 4.517%/year, and face no transactions costs, you can cash-and-carry arbitrage.

The FinancialCAD function aaCDF-repo can be used to compute the annualized implied repo rate for a forward contract on a commodity. See Figure 5.4.

The best borrowing rate available to market participants is the rate that corporations, dealers, institutions, and so on can obtain by engaging in what is called a **repurchase agreement**, or repo. In a repurchase agreement, party A owns some securities. He sells them to party B for a sum of money and agrees to repurchase them at a later date for a somewhat greater sum of money. Thus, A has effectively used the securities as collateral to borrow money from B. The interest rate associated with a repo is called the **repo rate**. Most repos are overnight loans, and sometimes the phrase **overnight repo rate** is used to describe the annualized borrowing rate for individuals who use repurchase agreements for one day. Repos of longer duration are called **term repos**.

aaCDF_repo		
spot price per unit of underlying commodity	58.25	
futures price	58.5	
value (settlement) date	24-Sep-2000	
expiry date	23-Dec-2000	
accrual method	2	actual/360
storage cost	0	
convenience value	0.4078	
statistic	1	annual repo rate incl. convenience value
annual repo rate incl. convenience value	**0.045170815**	

Figure 5.4 The FinancialCAD function aaCDF_repo computes the annualized implied repo rate for a forward contract on a commodity.

EXAMPLE 5.3 Party A owns $1 million face value of Treasury bills that currently have a market value of $950,000. Party A needs cash, and she can raise the needed funds by entering a repurchase agreement. Party A sells the T-bills to party B for $945,000[7] and agrees to repurchase them tomorrow for $945,311. Thus the annualized interest rate on this repurchase agreement is[8]:

$$\frac{F-S}{S}\times\frac{360}{1} = h(0, 1\text{ day})\times\frac{360}{1} = \frac{311}{945{,}000}\times 360\ (= 0.0003291\times 360) = 11.848\%$$

Arbitrage is possible if the implied repo rate for a forward contract exceeds actual market repo rates. This is equivalent to saying that the effective lending rate available in the forward market by buying the spot good and selling a forward contract exceeds the arbitrageur's borrowing rate.

The **implied reverse repo rate** is the rate of return earned by selling short the spot good and buying a forward contract. These two transactions are part of the reverse cash-and-carry arbitrage. The implied reverse repo rate is effectively a borrowing rate because the short sale of the spot good will provide a cash inflow, just as would any borrowing transaction. Going long a forward contract locks in a delivery date cash outflow. Arbitrageurs will compare the implied reverse repo rate to their own riskless lending rates. Whenever possible, they will risklessly borrow at the implied reverse repo rate in the forward market and lend at the higher lending rate available to them in the securities or real goods markets.

Summarizing, individuals will engage in cash-and-carry arbitrage if the implied repo rate in a forward contract exceeds the market repo rates. Under these conditions, they will borrow money at the market repo rate, buy the deliverable asset, and sell a forward contract. Individuals will engage in reverse cash-and-carry arbitrage whenever the implied reverse repo rate in a forward contract is less than the riskless lending rate. Recall that a reverse cash-and-carry arbitrage requires the short sale of the underlying asset, the purchase of a forward contract, and the lending of the proceeds of the short sale.

When markets are perfect, the implied repo rate and the implied reverse repo rate are equal. However, if (a) the purchase price of the spot good (the asked price) is greater than its selling price (the bid price), (b) there is a bid–asked spread on the forward contract, and/or (c) there are transactions costs required to do the trades, then the implied repo rate (the lending rate available in the forward market) will be less than the implied reverse repo rate (the borrowing rate available in the forward market). These market imperfections increase the range of forward prices that can exist without permitting arbitrage.

5.1.6 Forward Prices of Commodities: The Convenience Yield

The cost-of-carry model invokes perfect markets assumptions to determine theoretical forward prices. In particular, it is assumed that the commodity can be sold short and that those who own the commodity have no reservations about selling it.

However, for most goods other than financial assets and gold, the cost-of-carry model can be used only to determine an **upper bound**. In other words, for agricultural, energy, and metal forwards, we can say that $F \leq S + CC - CR$. If the forward price is above $S + CC - CR$, one can perform cash-and-carry arbitrage by using borrowed funds to buy the good in the spot market, and selling forward contracts. The carry costs must also account for storing, insuring, and otherwise maintaining the physical commodity.

The model cannot be used to determine the existence of arbitrage opportunities when the cash good must be sold short to arbitrage. The reason for this lies in what makes financial commodities different from all other physical commodities such as agricultural commodities. Financial instruments such as bonds, foreign exchange, and stocks are used for investment purposes. They are not used as part of any production process. Thus if the forward price was too low, no one would be inconvenienced by selling, or selling short, the cash good (the debt instruments, stocks, or foreign exchange) and buying an underpriced forward contract. No one would miss these commodities; they would be replaced on the delivery date of the forward contract. Indeed, individuals who arbitrage would be better off on the delivery date because they had sold these assets short (pure arbitrage) or out of their inventory (quasi-arbitrage).

We call goods that *are* used for production purposes noncarry commodities. The users of noncarry commodities will not always be willing to sell their inventory in the spot market and buy forward contracts to replenish their supplies at a later date. Similarly, potential pure arbitrageurs will not always be able to find supplies of the physical good to borrow and sell short. Producers need to maintain supplies of the goods that are used in their production processes. Cereal manufacturers need wheat and corn in inventory; electrical equipment manufacturers need supplies of copper to manufacture their products. Consequently, the cash prices of these noncarry commodities can even be above the forward price, and it is possible that no one will be willing and able to arbitrage by selling the good in the spot market and buying forward contracts.

The cost-of-carry formula can be modified to account for this situation by defining a new term, **convenience yield** (also sometimes called the convenience return). A commodity's convenience yield is the benefit, in dollars, that a user realizes for carrying inventory of the physical good above his or her immediate near-term needs. An oil refiner receives the convenience yield on crude oil inventories because without it, the refiner cannot produce any finished products.

Financial assets have no convenience return. In contrast, agricultural commodities frequently have a high convenience return because producers face losses if they have none of the commodity in inventory. A commodity's convenience return can be different for different users, and it can vary over time.[9] It will be at its highest when there are spot shortages of the cash good. At these times, spot prices will lie above forward prices.

Thus, for noncarry commodities, the cost-of-carry formula may be presented as follows;

$$F = S + CC - CR - \text{convenience return} \tag{5.5}$$

The problem with this formula is that the convenience return is not easily measured. Some users may have a low convenience return, and they will arbitrage when the forward price lies below $S + CC - CR - \textit{their}$ convenience return. Other users are indifferent between selling their supplies (or lending them to short sellers) and maintaining their inventory to realize their convenience return. In such cases, the relevant convenience return in Equation 5.5 is the marginal convenience return for that marginal producer. Finally, other producers will have a high convenience return, and they will never wish to sell or lend their supplies of the physical good.

Whenever the cash price of a deliverable commodity is above the forward price, a spot shortage (or temporary excess demand) of the good probably exists, and there is a high marginal convenience return.

5.1.7 Do Forward Prices Equal Expected Future Spot Prices?

5.1.7.1 Financial Assets

The cost-of-carry model determines the theoretical forward prices of financial assets such as interest rates, currencies, and stocks. Under the appropriate assumptions, actual forward prices must equal theoretical forward prices, or else arbitrage opportunities will exist. Expectations of future spot prices are not directly included in the forward prices of these assets. Any expectations of future spot prices are incorporated into the current spot price of the good, and given that spot price, as well as carry costs and carry returns, the forward price is found by the cost-of-carry model.

For example, if all investors expect the stock market to greatly appreciate in value in the near future, then the *spot* prices of stocks will rise to reflect those expectations. No matter how bullish investors are, the forward price of stocks must equal the spot price plus the net carrying costs (interest on the stocks minus the dividends between today and the delivery date).[10] Whether the forward price incorporates the market's expectation of the future spot price of the underlying commodity becomes a matter of semantics. The spot price reflects market expectations, and given the spot price and the cost-of-carry formula, the forward price is determined. The forward price incorporates expectations only because they are reflected in spot prices.

Therefore, the forward price of a financial asset or gold will rarely equal the market's expectation of the future spot price. Suppose that the spot price of gold is $280 and the interest rate is 0%. Then, the forward price must also equal $280, regardless of market expectations about the future spot price of gold.

Recall that market imperfections allow the forward price to lie in a band, or range, of prices without allowing for arbitrage opportunities. In these cases, the forward price will provide more information about market expectations. For example, if the theoretical forward price F is $350 \leq F \leq 352$ and investors are bullish, we would expect to see $F = 352$. If investors are bearish, then F will likely equal 350. Now, forward prices reveal market expectations, and futures prices are theoretically correct.

If F rises above 352, traders will arbitrage by buying the spot good and selling (overvalued) forward contracts. When the spot good is bought, its price will rise; when it is sold forward, the forward price will fall. Eventually, equilibrium will be reestablished, and there will be no opportunities to arbitrage. If F declines below 350, forwards will be bought and the spot good will be sold short.

The situation is different for the forward prices of physical commodities, where the forward price more likely reflects expected supply and expected demand at the delivery date. As such, the forward price does reflect the market's expectation of the future spot price. But it may not equal the expected spot price. There are theories that predict the possible existence of risk premiums in forward prices. A risk premium is defined to be the difference between the forward price and market's expectation of what the spot price will be at delivery. The theories give rise to the concepts of *normal backwardation* and *contango*.[11]

5.1.7.2 Physical Commodities: Normal Backwardation and Contango

In a world of certainty, determining the forward price of any commodity is easy. It will equal the future spot price of the good at delivery, which all investors know with certainty. There is nothing to prevent the spot price from changing in a known, perfectly predictable manner, such that the forward price lies above or below the current spot price. The spot price will fluctuate according to supply and demand conditions. However, all investors know what the spot price will be at every subsequent date, including the delivery date of the forward contract. The spot price that all investors know will exist on the delivery date equals the forward price, and in a world of certainty, the forward price will never change. Buyers and sellers of forward contracts put up no money to assume their positions, and they will realize no subsequent cash flows from any changes in forward prices. Speculators, hedgers, and arbitrageurs have no reason to exist because the spot price at delivery is known with certainty and the forward price is constant.

While the situation is different under conditions of uncertainty, many still believe in the **unbiased expectations hypothesis of forward prices**. According to this theory, the forward price equals the market's expectation of what the spot price will be at delivery:

$$F_t = E_t(\tilde{S}_T)$$

This formulation states that F_t, the forward price at any time t, equals the time t expectation of what the spot price of the commodity will be at delivery (time T). Under this hypothesis, the expected cash flow to any forward position is zero; today's forward price equals the expected forward price on the delivery day. And it should be obvious that the expected forward price for delivery at time T equals the expected spot price on day T.

If the unbiased expectations hypothesis of forward prices is correct, then any forward price is determined because traders have used all the information available to come to the aggregate expectation that on the delivery date, the spot price for the good will be that forward price. In other words, $F_t = E_t(\tilde{S}_T)$.

If speculators are risk neutral, the unbiased expectations theory is likely to explain commodity forward prices. Risk neutrality means that all assets are priced to provide the same riskless rate of return. If any asset was priced to yield an expected return greater than the riskless interest rate, investors would buy it, regardless of the asset's riskiness. Similarly, no investor would ever buy an asset if he thought it would yield a return less than the riskless rate. Forward positions, long and short, can be entered into with no cash outlay. In a risk-neutral world, therefore, the expected cash flow to both longs and shorts must be zero.

Others, however, believe that the forward price is a biased predictor of what the future spot price will be. Keynes (1930) first proposed that forward prices will contain a risk premium. He hypothesized that hedgers tend to sell forward contracts and futures contracts. If hedgers are net short, then it must follow that speculators have more long positions than short positions.[12] But speculators will assume long positions only if they expect to earn a higher return, a risk premium. Therefore, the forward price must be below the spot price that investors expect to prevail at delivery, $F_t < E(\tilde{S}_T)$. The forward price is expected to rise over the life of the contract, to provide a positive expected return to speculators. Risk-averse hedgers are willing to lose a little to shed the price risks they face in an unhedged position.

Keynes called this situation **normal backwardation**. According to the normal backwardation hypothesis, forward prices are below expected future spot prices, and the forward price is expected to rise as the delivery date approaches. The reason for backwardation is that hedgers are usually short forward. A hypothetical price process of a forward contract exhibiting normal backwardation is shown in Figure 5.5.[13]

A **contango** exists whenever the forward price lies above the expected future spot price, $F_t > E_t(\tilde{S}_T)$. Here, the forward price should generally decline as the delivery date nears. This situation arises because for some commodities and at some times, hedgers will be net long forward contracts, and speculators will be net sellers of forward contracts. Speculators must expect to earn a profit if they are to sell forwards. In a contango, forward prices must be expected to fall. A contango is shown in Figure 5.6.

Cootner's (1960) theory about the seasonality of hedging activities can be modified to present a summary statement about the issue of whether forward prices equal expected future spot prices

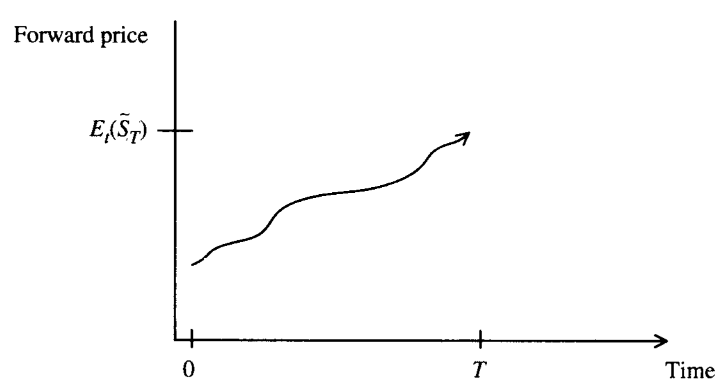

Figure 5.5 Normal backwardation. The forward price will lie below the existing expectation (as of any time t prior to the delivery date, time T) of what the time T (delivery date) spot price will be. The forward price will rise, on average. It is assumed that $E_t(S_T)$ remains unchanged as the delivery date nears.

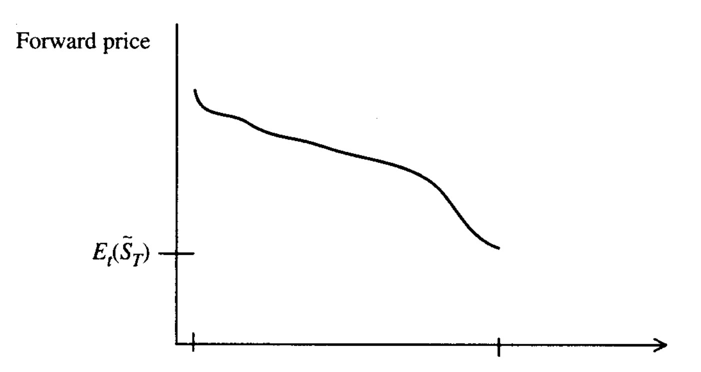

Figure 5.6 Contango. The forward price lies above the existing expectation (as of any time t prior to the delivery date, time T) of what the time T (delivery date) spot price will be. The forward price will fall, on average. It is assumed that $E_t(\tilde{S}_T)$ remains unchanged as the delivery date nears.